Semantics and Verification of Software

Lecture 15: Dataflow Analysis II
(Available Expressions & Live Variables)

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Outline

1. Repetition: Dataflow Analysis

2. Live Variables Analysis

3. Heading for a Dataflow Analysis Framework

Labelled Programs

- Goal: localization of analysis information
- Dataflow information will be associated with
  - `skip` statements
  - assignments
  - tests in conditionals (`if`) and loops (`while`)

  These constructs will be called blocks.
- Assume set of labels L with meta variable $l \in L$ (usually $L = \mathbb{N}$)

Definition (Labelled WHILE programs)

The syntax of labelled WHILE programs is defined by the following context-free grammar:

$$
\begin{aligned}
a &::= z \mid x \mid a_1\texttt{+}a_2 \mid a_1\texttt{-}a_2 \mid a_1\texttt{*}a_2 \in AExp\\
b &::= t \mid a_1\texttt{=}a_2 \mid a_1\texttt{>}a_2 \mid \neg b \mid b_1 \wedge b_2 \mid b_1 \vee b_2 \in BExp\\
c &::= [\texttt{skip}]^l \mid [x \texttt{ := } a]^l \mid c_1\texttt{;}c_2 \mid\\
&\qquad \texttt{if } [b]^l \texttt{ then } c_1 \texttt{ else } c_2 \mid \texttt{while } [b]^l \texttt{ do } c \in Cmd
\end{aligned}
$$

Here all labels in a statement $c \in Cmd$ are assumed to be distinct.

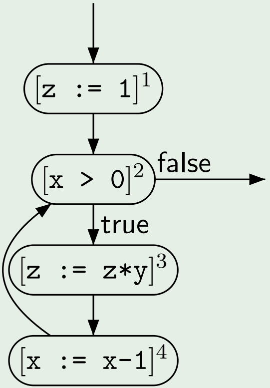
Representing Control Flow

Example

$c = [$`z := 1`$]^1$;
    `while` $[$`x > 0`$]^2$ `do`
        $[$`z := z*y`$]^3$;
        $[$`x := x-1`$]^4$

$\mathsf{init}(c) = 1$
$\mathsf{final}(c) = \{2\}$
$\mathsf{flow}(c) = \{(1,2), (2,3), (3,4), (4,2)\}$

Visualization by flow graph:



Goal of Available Expressions Analysis

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The goal of Available Expressions Analysis is to determine, for each program point, which (complex) expressions *must* have been computed, and not later modified, on all paths to the program point.

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- only interesting for non-trivial (i.e., complex) arithmetic expressions

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Example (Available Expressions Analysis)

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[x := a+b]^1;
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while [y > a+b]^3 do
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- a+b available at label 3

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[x := a+b]^1;
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```

- `a+b` available at label 3
- `a+b` not available at label 5
- possible optimization: `while [y > x]^3 do`

The Equation System

Reminder:

$$\mathsf{AE}_l = \begin{cases} \emptyset & \text{if } l = \mathsf{init}(c) \\ \bigcap\{\varphi_{l'}(\mathsf{AE}_{l'}) \mid (l', l) \in \mathsf{flow}(c)\} & \text{otherwise} \end{cases}$$

$$\varphi_{l'}(E) = (E \setminus \mathsf{kill}_{\mathsf{AE}}(B^{l'})) \cup \mathsf{gen}_{\mathsf{AE}}(B^{l'})$$

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$c =$ [x := a+b]^1;
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$l \in L_c$	$\mathsf{kill}_{\mathsf{AE}}(B^l)$	$\mathsf{gen}_{\mathsf{AE}}(B^l)$
1	$\emptyset$	{a+b}
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4	{a+b, a*b, a+1}	$\emptyset$
5	$\emptyset$	{a+b}

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Equations:

$$\begin{aligned}
\mathsf{AE}_1 &= \emptyset \\
\mathsf{AE}_2 &= \varphi_1(\mathsf{AE}_1) = \mathsf{AE}_1 \cup \{\mathtt{a+b}\} \\
\mathsf{AE}_3 &= \varphi_2(\mathsf{AE}_2) \cap \varphi_5(\mathsf{AE}_5) \\
&= (\mathsf{AE}_2 \cup \{\mathtt{a*b}\}) \cap (\mathsf{AE}_5 \cup \{\mathtt{a+b}\}) \\
\mathsf{AE}_4 &= \varphi_3(\mathsf{AE}_3) = \mathsf{AE}_3 \cup \{\mathtt{a+b}\} \\
\mathsf{AE}_5 &= \varphi_4(\mathsf{AE}_4) = \mathsf{AE}_4 \setminus \{\mathtt{a+b}, \mathtt{a*b}, \mathtt{a+1}\}
\end{aligned}$$

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while [y > a+b]^3 do
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Solution:

$$\begin{aligned} \mathsf{AE}_1 &= \emptyset \\ \mathsf{AE}_2 &= \{\texttt{a+b}\} \\ \mathsf{AE}_3 &= \{\texttt{a+b}\} \\ \mathsf{AE}_4 &= \{\texttt{a+b}\} \\ \mathsf{AE}_5 &= \emptyset \end{aligned}$$

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The goal of Live Variables Analysis is to determine, for each program point, which variables *may* be live at the exit from the point.

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- All variables considered to be live at the end of the program (alternative: restriction to output variables)

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- A variable is called live at the exit from a block if there exists a path from the block to a use of the variable that does not re-define the variable
- All variables considered to be live at the end of the program (alternative: restriction to output variables)
- Can be used for Dead Code Elimination:
  remove assignments to non-live variables

An Example

Example 15.1 (Live Variables Analysis)

$[\texttt{x := 2}]^1$;
$[\texttt{y := 4}]^2$;
$[\texttt{x := 1}]^3$;
if $[\texttt{y > 0}]^4$ **then**
  $[\texttt{z := x}]^5$
else
  $[\texttt{z := y*y}]^6$;
$[\texttt{x := z}]^7$

An Example

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- x not live at exit from label 1

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- x not live at exit from label 1
- y live at exit from 2

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- y live at exit from 2
- x live at exit from 3

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- x not live at exit from label 1
- y live at exit from 2
- x live at exit from 3
- z live at exits from 5 and 6

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```

- x not live at exit from label 1
- y live at exit from 2
- x live at exit from 3
- z live at exits from 5 and 6
- possible optimization: remove [x := 2]^1

Formalizing Live Variables Analysis I

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$$\mathsf{kill}_{\mathsf{LV}}([\texttt{skip}]^l) := \emptyset$$
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- Every reading access generates a live variable
- Formally: $\mathsf{gen}_{\mathsf{LV}} : Block_c \to 2^{Var_c}$ is defined by

$$\begin{aligned}\mathsf{gen}_{\mathsf{AE}}([\mathtt{skip}]^l) &:= \emptyset\\ \mathsf{gen}_{\mathsf{AE}}([x \mathtt{\ :=\ } a]^l) &:= FV(a)\\ \mathsf{gen}_{\mathsf{AE}}([b]^l) &:= FV(b)\end{aligned}$$

Example 15.2 ($\mathsf{kill}_{\mathsf{LV}}/\mathsf{gen}_{\mathsf{LV}}$ functions)

$c =$ [x := 2]^1;
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- $Var_c = \{\mathtt{x}, \mathtt{y}, \mathtt{z}\}$

Formalizing Live Variables Analysis II

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$$\mathsf{LV}_l = \begin{cases} Var_c & \text{if } l \in \mathsf{final}(c) \\ \bigcup\{\varphi_{l'}(\mathsf{LV}_{l'}) \mid (l,l') \in \mathsf{flow}(c)\} & \text{otherwise} \end{cases}$$
where $\varphi_{l'} : 2^{Var_c} \to 2^{Var_c}$ denotes the transfer function of block $B^{l'}$, given by
$$\varphi_{l'}(V) := (V \setminus \mathsf{kill}_{\mathsf{LV}}(B^{l'})) \cup \mathsf{gen}_{\mathsf{LV}}(B^{l'})$$

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- Characterization of analysis:

backward: starts in $\mathsf{final}(c)$ and proceeds upwards
may: $\bigcup$ in equation for LV_l
flow-sensitive: results depending on order of assignments

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- Characterization of analysis:

  backward: starts in $\mathsf{final}(c)$ and proceeds upwards

  may: $\bigcup$ in equation for LV_l

  flow-sensitive: results depending on order of assignments
- Later: solution not necessarily unique

  $\implies$ choose least one

The Equation System II

Reminder:

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$$\begin{aligned}
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\mathsf{LV}_2 &= \varphi_3(\mathsf{LV}_3) = \mathsf{LV}_3 \setminus \{\mathtt{x}\} \\
\mathsf{LV}_3 &= \varphi_4(\mathsf{LV}_4) = \mathsf{LV}_4 \cup \{\mathtt{y}\} \\
\mathsf{LV}_4 &= \varphi_5(\mathsf{LV}_5) \cup \varphi_6(\mathsf{LV}_6) \\
&= ((\mathsf{LV}_5 \setminus \{\mathtt{z}\}) \cup \{\mathtt{x}\}) \cup ((\mathsf{LV}_6 \setminus \{\mathtt{z}\}) \cup \{\mathtt{y}\}) \\
\mathsf{LV}_5 &= \varphi_7(\mathsf{LV}_7) = (\mathsf{LV}_7 \setminus \{\mathtt{x}\}) \cup \{\mathtt{z}\} \\
\mathsf{LV}_6 &= \varphi_7(\mathsf{LV}_7) = (\mathsf{LV}_7 \setminus \{\mathtt{x}\}) \cup \{\mathtt{z}\} \\
\mathsf{LV}_7 &= \{\mathtt{x},\mathtt{y},\mathtt{z}\}
\end{aligned}$$

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7	$\{x\}$	$\{z\}$

$$\begin{aligned}
\mathsf{LV}_1 &= \varphi_2(\mathsf{LV}_2) = \mathsf{LV}_2 \setminus \{y\} \\
\mathsf{LV}_2 &= \varphi_3(\mathsf{LV}_3) = \mathsf{LV}_3 \setminus \{x\} \\
\mathsf{LV}_3 &= \varphi_4(\mathsf{LV}_4) = \mathsf{LV}_4 \cup \{y\} \\
\mathsf{LV}_4 &= \varphi_5(\mathsf{LV}_5) \cup \varphi_6(\mathsf{LV}_6) \\
&= ((\mathsf{LV}_5 \setminus \{z\}) \cup \{x\}) \cup ((\mathsf{LV}_6 \setminus \{z\}) \cup \{y\}) \\
\mathsf{LV}_5 &= \varphi_7(\mathsf{LV}_7) = (\mathsf{LV}_7 \setminus \{x\}) \cup \{z\} \\
\mathsf{LV}_6 &= \varphi_7(\mathsf{LV}_7) = (\mathsf{LV}_7 \setminus \{x\}) \cup \{z\} \\
\mathsf{LV}_7 &= \{x, y, z\}
\end{aligned}$$

Solution:

$$\begin{aligned}
\mathsf{LV}_1 &= \emptyset \\
\mathsf{LV}_2 &= \{y\} \\
\mathsf{LV}_3 &= \{x, y\} \\
\mathsf{LV}_4 &= \{x, y\} \\
\mathsf{LV}_5 &= \{y, z\} \\
\mathsf{LV}_6 &= \{y, z\} \\
\mathsf{LV}_7 &= \{x, y, z\}
\end{aligned}$$

Outline

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- **Advantage:** possibility for designing (efficient) generic algorithms for solving dataflow equations
- **Overall pattern:** for $c \in Cmd$ and $l \in L_c$, the analysis information (AI) is described by equations of the form

$$\mathsf{AI}_l = \begin{cases} \iota & \text{if } l \in E \\ \bigsqcup\{\varphi_{l'}(\mathsf{AI}_{l'}) \mid (l', l) \in F\} & \text{otherwise} \end{cases}$$

where

- ι specifies the initial analysis information
- E is $\{\mathsf{init}(c)\}$ or $\mathsf{final}(c)$
- $\bigsqcup$ is $\bigcap$ or $\bigcup$
- $\varphi_{l'}$ denotes the transfer function of block $B^{l'}$
- F is $\mathsf{flow}(c)$ or $\mathsf{flow}^R(c)$ $(:= \{(l', l) \mid (l, l') \in \mathsf{flow}(c)\})$

Characterization of Analyses

- **Direction of information flow:**
  - forward:
    - $F = \mathsf{flow}(c)$
    - AI_l concerns entry of B^l
    - c has isolated entry
  - backward:
    - $F = \mathsf{flow}^R(c)$
    - AI_l concerns exit of B^l
    - c has isolated exits

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    - c has isolated exits
- **Quantification over paths:**
  - may:
    - $\bigsqcup = \bigcup$
    - property satisfied by some path
    - interested in least solution (later)
  - must:
    - $\bigsqcup = \bigcap$
    - property satisfied by all paths
    - interested in greatest solution (later)