

4. Exercise sheet *Semantics and Verification of Software SoSe2010*

Due to Monday, 17th May 2010, *before* the exercise course begins.

Exercise 4.1: (3 points)

Show that the operational and the denotational semantics of Boolean expressions coincide, i.e. prove the following result.

For every $b \in \mathbf{BExp}$, $\sigma \in \Sigma$, and $t \in \mathbb{B}$:

$$\langle b, \sigma \rangle \rightarrow t \quad \text{iff} \quad \mathfrak{A}[b](\sigma) = t.$$

You may assume we already know that the operational and denotational semantics of arithmetic expressions coincide, thus fulfilling:

For every $a \in \mathbf{AExp}$, $\sigma \in \Sigma$, and $z \in \mathbb{Z}$:

$$\langle a, \sigma \rangle \rightarrow z \quad \text{iff} \quad \mathfrak{A}[a](\sigma) = z.$$

Exercise 4.2: (2+2 points)

Consider the following fragment of the factorial program (see Exercise 3.1):

while $\neg(x = 1)$ **do** $(y := y * x; x := x - 1)$.

- (a) Determine the corresponding functional $\Phi : (\Sigma \dashrightarrow \Sigma) \rightarrow (\Sigma \dashrightarrow \Sigma)$.
- (b) Give at least two fixpoints of Φ .

Exercise 4.3: (2+1 points)

Given a function $f : \Sigma \dashrightarrow \Sigma$ transforming a program state σ for fixed $n, m \in \mathbb{N}$, $m > n > 1$ into:

$$f(\sigma) = \begin{cases} \sigma[x \rightarrow 0] & \text{if } \sigma(x) = 1, \dots, n-1 \\ \sigma[x \rightarrow 1] & \text{if } \sigma(x) = 0 \\ \sigma[x \rightarrow \sigma(x) - 1] & \text{if } n \leq \sigma(x) \leq m \\ \perp & \text{otherwise} \end{cases}$$

- (a) Sketch $graph(f)$ in set as well as in graphical representation.
- (b) Is $\Phi : (\Sigma \dashrightarrow \Sigma) \rightarrow (\Sigma \dashrightarrow \Sigma)$ given by $\Phi(f) = f$ monotonic with respect to the partial order \sqsubseteq given by graph inclusion as defined in the lecture?

Exercise 4.4: (1 point)

Show that $f \sqsubseteq g$ is equivalent to requiring $graph(f) \subseteq graph(g)$ (both defined in the lecture)?