

## 4. Exercise sheet *Semantics and Verification of Software SoSe2010*

Due to Monday, 17th May 2010, *before* the exercise course begins.

### Exercise 4.1:

(3 points)

Show that the operational and the denotational semantics of Boolean expressions coincide, i.e. prove the following result.

For every  $b \in \mathbf{BExp}$ ,  $\sigma \in \Sigma$ , and  $t \in \mathbb{B}$ :

$$\langle b, \sigma \rangle \rightarrow t \quad \text{iff} \quad \mathcal{A}[[b]](\sigma) = t.$$

You may assume we already know that the operational and denotational semantics of arithmetic expressions coincide, thus fulfilling:

For every  $a \in \mathbf{AExp}$ ,  $\sigma \in \Sigma$ , and  $z \in \mathbb{Z}$ :

$$\langle a, \sigma \rangle \rightarrow z \quad \text{iff} \quad \mathcal{A}[[a]](\sigma) = z.$$

### Exercise 4.2:

(2+2 points)

Consider the following fragment of the factorial program (see Exercise 3.1):

**while**  $\neg(x = 1)$  **do**  $(y := y * x; x := x - 1).$

- (a) Determine the corresponding functional  $\Phi : (\Sigma \rightarrow \Sigma) \rightarrow (\Sigma \rightarrow \Sigma)$ .
- (b) Give at least two fixpoints of  $\Phi$ .

### Exercise 4.3:

(2+1 points)

Given a function  $f : \Sigma \rightarrow \Sigma$  transforming a program state  $\sigma$  for fixed  $n, m \in \mathbb{N}$ ,  $m > n > 1$  into:

$$f(\sigma) = \begin{cases} \sigma[x \rightarrow 0] & \text{if } \sigma(x) = 1, \dots, n-1 \\ \sigma[x \rightarrow 1] & \text{if } \sigma(x) = 0 \\ \sigma[x \rightarrow \sigma(x) - 1] & \text{if } n \leq \sigma(x) \leq m \\ \perp & \text{otherwise} \end{cases}$$

- (a) Sketch  $\text{graph}(f)$  in set as well as in graphical representation.
- (b) Is  $\Phi : (\Sigma \rightarrow \Sigma) \rightarrow (\Sigma \rightarrow \Sigma)$  given by  $\Phi(f) = f$  monotonic with respect to the partial order  $\sqsubseteq$  given by graph inclusion as defined in the lecture?

### Exercise 4.4:

(1 point)

Show that  $f \sqsubseteq g$  is equivalent to requiring  $\text{graph}(f) \subseteq \text{graph}(g)$  (both defined in the lecture)?