

5. Exercise sheet *Semantics and Verification of Software SoSe2010*

Due to Monday, 31th May 2010, *before* the exercise course begins.

Exercise 5.1: (3 points)

Consider the class of binary numerals, i.e. numeric values of the binary numeral system using only the two values, 0 and 1.

- (a) Give a grammar generating arbitrary binary numerals.
- (b) Extend the syntax of binary numerals in order to be able to handle signed binary numerals.
- (c) Specify the complete denotational semantic functional for the new constructs, where denotations should reflect the decimal value of the numerals.

Exercise 5.2: (2 points)

Show that the following propositions hold:

- (a) The identity function id_D on a ccpo D is continuous.
- (b) Let $f : D \rightarrow E$ and $g : E \rightarrow F$ be continuous functions on ccpos D, E, F . Then their composition $g \circ f : D \rightarrow F$ is continuous.

Exercise 5.3: (3 points)

Investigate

$$\mathbb{C} \llbracket z := 0; \textbf{while } x > 0 \textbf{ do } (x := x - 1; z := z + y) \rrbracket$$

in analogy to the factorial example 8.9 given in lecture 8.

Exercise 5.4: (3 points)

- (a) Define the denotational semantics of the **repeat** c **until** b construct.
- (b) Using this semantics, show that the following semantic equivalence holds:

$$\textbf{repeat } c \textbf{ until } b \sim c; \textbf{while } \neg b \textbf{ do } c.$$

(**Hint:** The proof can be given by complete induction over the fixpoint iteration index n .)