

## 5. Exercise sheet *Semantics and Verification of Software SoSe2010*

Due to Monday, 31th May 2010, *before* the exercise course begins.

### Exercise 5.1:

(3 points)

Consider the class of binary numerals, i.e. numeric values of the binary numeral system using only the two values, 0 and 1.

- (a) Give a grammar generating arbitrary binary numerals.
- (b) Extend the syntax of binary numerals in order to be able to handle signed binary numerals.
- (c) Specify the complete denotational semantic functional for the new constructs, where denotations should reflect the decimal value of the numerals.

### Exercise 5.2:

(2 points)

Show that the following propositions hold:

- (a) The identity function  $id_D$  on a ccpo  $D$  is continuous.
- (b) Let  $f : D \rightarrow E$  and  $g : E \rightarrow F$  be continuous functions on ccpo's  $D, E, F$ . Then their composition  $g \circ f : D \rightarrow F$  is continuous.

### Exercise 5.3:

(3 points)

Investigate

$$\mathfrak{C}[\![z := 0; \mathbf{while } x > 0 \mathbf{ do } (x := x - 1; z := z + y)]\!]$$

in analogy to the factorial example 8.9 given in lecture 8.

### Exercise 5.4:

(3 points)

- (a) Define the denotational semantics of the **repeat**  $c$  **until**  $b$  construct.
- (b) Using this semantics, show that the following semantic equivalence holds:

$$\mathbf{repeat } c \mathbf{ until } b \sim c; \mathbf{while } \neg b \mathbf{ do } c.$$

(**Hint:** The proof can be given by complete induction over the fixpoint iteration index  $n$ .)