

Semantics and Verification of Software

Lecture 15: Semantics of Blocks and Procedures

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- 1 Summary: Axiomatic Semantics
- 2 Extension by Blocks and Procedures
- 3 Operational Semantics of Blocks and Procedures

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- **Equivalence** of axiomatic and operational/denotational semantics
- **Software engineering** aspect: integrated development of program and proof (cf. assertions in Java)

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- Important: **scope** of variable and procedure identifiers
 - static scoping: scope of identifier = **declaration environment** (here)
 - dynamic scoping: scope of identifier = **calling environment** (old Algol/Lisp dialects)

Example 15.1

```
begin
  var x; var y;
  proc P is y := x;
  x := 1;
  begin
    var x;
    x := 2;
    call P
  end
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Static and Dynamic Scoping

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static scoping \Rightarrow $y = 1$

dynamic scoping \Rightarrow $y = 2$

Extending the Syntax

Syntactic categories:

Category	Domain	Meta variable
Procedure identifiers	$PVar = \{P, Q, \dots\}$	P
Procedure declarations	$PDec$	p
Variable declarations	$VDec$	v
Commands (statements)	Cmd	c

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Context-free grammar:

$$p ::= \text{proc } P \text{ is } c; p \mid \varepsilon \in PDec$$
$$v ::= \text{var } x; v \mid \varepsilon \in VDec$$
$$c ::= \text{skip} \mid x := a \mid c_1; c_2 \mid \text{if } b \text{ then } c_1 \text{ else } c_2 \mid \text{while } b \text{ do } c \mid \text{call } P \mid \text{begin } v p c \text{ end} \in Cmd$$

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- Now: explicit control over all (nested) **instances** of a variable:
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- ① determine current memory location of x :

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- Thus: previous **state** information represented as $\sigma \circ \rho$

- Effect of procedure call determined by its body statement and variable and procedure environment of its declaration:

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- Effect of declaration: update of environment

$$\text{upd}_v[\cdot] : VDec \times VEnv \times Sto \rightarrow VEnv \times Sto$$

$$\begin{aligned}\text{upd}_v[\text{var } x; v](\rho, \sigma) &:= \text{upd}_v[v](\rho[x \mapsto l_x], \sigma[l_x \mapsto 0]) \\ \text{upd}_v[\varepsilon](\rho, \sigma) &:= (\rho, \sigma)\end{aligned}$$

$$\text{upd}_p[\cdot] : PDec \times VEnv \times PEnv \rightarrow PEnv$$

$$\begin{aligned}\text{upd}_p[\text{proc } P \text{ is } c; p](\rho, \pi) &:= \text{upd}_p[p](\rho, \pi[P \mapsto (c, \rho, \pi)]) \\ \text{upd}_p[\varepsilon](\rho, \pi) &:= \pi\end{aligned}$$

where $l_x := \min\{l \in \mathbb{N} \mid \sigma(l) = \perp\}$

Definition 15.2 (Execution relation)

For $c \in Cmd$, $\sigma, \sigma' \in Sto$, $\rho \in VEnv$, and $\pi \in PEnv$, the **execution relation** $(\rho, \pi) \vdash \langle c, \sigma \rangle \rightarrow \sigma'$ is defined by the following rules:

$$(\text{skip}) \frac{}{(\rho, \pi) \vdash \langle \text{skip}, \sigma \rangle \rightarrow \sigma}$$

$$(\text{asgn}) \frac{\langle a, \sigma \circ \rho \rangle \rightarrow z}{(\rho, \pi) \vdash \langle x := a, \sigma \rangle \rightarrow \sigma[\rho(x) \mapsto z]}$$

$$(\text{seq}) \frac{(\rho, \pi) \vdash \langle c_1, \sigma \rangle \rightarrow \sigma' \quad (\rho, \pi) \vdash \langle c_2, \sigma' \rangle \rightarrow \sigma''}{(\rho, \pi) \vdash \langle c_1 ; c_2, \sigma \rangle \rightarrow \sigma''}$$

$$(\text{if-t}) \frac{\langle b, \sigma \circ \rho \rangle \rightarrow \text{true} \quad (\rho, \pi) \vdash \langle c_1, \sigma \rangle \rightarrow \sigma'}{(\rho, \pi) \vdash \langle \text{if } b \text{ then } c_1 \text{ else } c_2, \sigma \rangle \rightarrow \sigma'}$$

$$(\text{if-f}) \frac{\langle b, \sigma \circ \rho \rangle \rightarrow \text{false} \quad (\rho, \pi) \vdash \langle c_2, \sigma \rangle \rightarrow \sigma'}{(\rho, \pi) \vdash \langle \text{if } b \text{ then } c_1 \text{ else } c_2, \sigma \rangle \rightarrow \sigma'}$$

Definition 15.2 (Execution relation; continued)

$$(\text{wh-f}) \frac{\langle b, \sigma \circ \rho \rangle \rightarrow \text{false}}{(\rho, \pi) \vdash \langle \text{while } b \text{ do } c, \sigma \rangle \rightarrow \sigma}$$

$$(\text{wh-t}) \frac{\langle b, \sigma \circ \rho \rangle \rightarrow \text{true } (\rho, \pi) \vdash \langle c, \sigma \rangle \rightarrow \sigma' \ (\rho, \pi) \vdash \langle \text{while } b \text{ do } c, \sigma' \rangle \rightarrow \sigma''}{(\rho, \pi) \vdash \langle \text{while } b \text{ do } c, \sigma \rangle \rightarrow \sigma''}$$

$$(\text{call}) \frac{(\rho', \pi'[P \mapsto (c, \rho', \pi')]) \vdash \langle c, \sigma \rangle \rightarrow \sigma'}{(\rho, \pi) \vdash \langle \text{call } P, \sigma \rangle \rightarrow \sigma'} \quad \text{if } \pi(P) = (c, \rho', \pi')$$

$$(\text{block}) \frac{\text{upd}_v[v](\rho, \sigma) = (\rho', \sigma') \quad (\rho', \text{upd}_p[p](\rho', \pi)) \vdash \langle c, \sigma' \rangle \rightarrow \sigma''}{(\rho, \pi) \vdash \langle \text{begin } v \text{ } p \text{ } c \text{ end}, \sigma \rangle \rightarrow \sigma''}$$

Remarks about rules (call) and (block):

- **Static scoping** is modelled in (call) by using the environments ρ' and π' (as determined in (block)) from the declaration site of procedure P (and not ρ and π from the calling site)
- In (call), the procedure environment associated with procedure P is extended by a P -entry to handle **recursive calls** of P :

$$\pi'[P \mapsto (c, \rho', \pi')]$$

Example 15.3

```

c = begin
    var x; var y; } v
    proc F is
        begin
            var z;
            z := x;
            if z=1 then skip
                else x := x-1;
            call F;
            y := z * y
        } c1
    end
    x := 2; y := 1; call F } c0
end
    } c2
} cF
} p
}

```

Let $\sigma_\emptyset(l) = \rho_\emptyset(x) = \pi_\emptyset(P) = \perp$ for all $l \in Loc, x \in Var, P \in PVar$

Notation: $\sigma_{ijkl} \Leftrightarrow \sigma(0) = i, \sigma(1) = j, \sigma(2) = k, \sigma(3) = l$

Derivation tree for $(\rho_\emptyset, \pi_\emptyset) \vdash \langle c, \sigma_\emptyset \rangle \rightarrow \sigma_{1221}$: on the board