Give a formal proof, using the operational semantics in Definition 12.4.

Lösung:

σ : **Loc** $\dashrightarrow \mathbb{Z}$ is representing the memory
ρ : **Var** $\dashrightarrow$ **Loc** is mapping variables to memory cells
π is mapping procedure names to code and declaration environment

Choose $\rho = \rho_\emptyset$. Initially the value of the memory is undefined.

The proof structure:

$$\dfrac{\dfrac{\dfrac{\overline{4}}{\sigma_0\ 3\ \sigma_1}\,\text{asgn} \qquad \dfrac{\dfrac{\dfrac{\overline{8}}{\sigma_1\ 7\ \sigma_2}\,\text{asgn} \qquad \dfrac{\dfrac{\overline{11}}{\sigma_2\ 10\ \sigma_2'}\,\text{asgn}}{\sigma_2\ 9\ \sigma_2'}\,\text{call}}{\sigma_1\ 6\ \sigma_2'}\,\text{seq}}{\sigma_1\ 5\ \sigma_2'}\,\text{block}}{\sigma_0\ 2\ \sigma_2'}\,\text{seq}}{\sigma_0\ 1\ \sigma_2'}\,\text{block}$$

[1] $(\rho_0, \pi_0) \vdash \langle c_0, \sigma_0 \rangle \to \sigma_2'$

block-rule updates the environment only:

[2] $(\rho_1, \pi_1) \vdash \langle c_1, \sigma_0[0 \mapsto 0][1 \mapsto 1] \rangle \to \sigma_2'$

where $\rho_1 := \rho_0[x \mapsto 0][y \mapsto 1]$ and $\pi_1 := \pi_0[P \mapsto (y := x, \rho_1, \pi_0)]$ $\quad \overset{\rho_1}{\sigma_0}$: $[\ \overset{x}{\cdot} \mid \overset{y}{\cdot} \mid \ldots$
and $c_1 \equiv x := 1;$ **begin** . . . **end**:

[3] $(\rho_1, \pi_1) \vdash \langle x := 1, \sigma_0[0 \mapsto 0][1 \mapsto 0] \rangle \to \sigma_1$ where $\sigma_1 = \sigma_0[0 \mapsto 1][1 \mapsto 0]$ since
[4] $\langle 1, \sigma_0[0 \mapsto 0][1 \mapsto 0] \circ \rho_1 \rangle \to 1$ $\quad \overset{\rho_1}{\sigma_1}$: $[\ \overset{x}{1} \mid \overset{y}{\cdot} \mid \ldots$

[5] $(\rho_1, \pi_1) \vdash \langle \textbf{begin} \ldots \textbf{end}, \sigma_1 \rangle \to \sigma_2'$

block-rule updates the environment only:

[6] $(\rho_2, \pi_2) \vdash \langle c_2, \sigma_1[2 \mapsto 0] \rangle \to \sigma_2'$

where $\rho_2 := \rho_1[x \mapsto 2]$ and $\pi_2 := \pi_1$ $\quad \overset{\rho_2}{\sigma_1}$: $[\ 1 \mid \overset{y}{\cdot} \mid \overset{x}{\cdot} \mid \ldots$
and $c_2 \equiv x := 2;$ **call** P:

[7] $(\rho_2, \pi_2) \vdash \langle x := 2, \sigma_1[2 \mapsto 0] \rangle \to \sigma_2$ where $\sigma_2 = \sigma_1[2 \mapsto 2]$ since
[8] $\langle 2, \sigma_1[2 \mapsto 0] \circ \rho_2 \rangle \to 2$ $\quad \overset{\rho_2}{\sigma_2}$: $[\ 1 \mid \overset{y}{\cdot} \mid \overset{x}{2} \mid \ldots$

[9] $(\rho_2, \pi_2) \vdash \langle \textbf{call } P, \sigma_2 \rangle \to \sigma_2'$

call-rule evaluates σ_2 to σ_2', if the procedure does so in its declaration environment (here ρ_1 and π_0):

[10] $(\rho_2, \overbrace{\pi_2'[P \mapsto (y := x, \rho_2, \pi_2')]}^{\pi_2'}) \vdash \langle y := x, \sigma_2 \rangle \to \sigma_2'$
. where $\sigma_2' = \sigma_2[1/1]$, $\pi_2(P) = (y := x, \rho_2, \pi_2')$ since **[11]** $\langle x, \sigma_2 \circ \rho_1 \rangle \to 1$ $\quad \overset{\rho_1}{\sigma_2'}$:
$[\ \overset{x}{1} \mid \overset{y}{1} \mid \overset{x_{loc}}{2} \mid \ldots$

We get that $\sigma_2' = \sigma_0[0 \mapsto 1, 1 \mapsto 1, 2 \mapsto 2]$.