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Exercise 1 (Programming Language WHILE): (1 + 1 Points)

a) Write a program using the WHILE programming language that calculates z := x div y with x,y,z $\in \mathbb{Z}$.

b) Draw the flow diagram for the program from part a).

Exercise 2 (Evaluation Relations): (1 + 4 Points)

a) Declare a rule for the arithmetic expression a_1 div a_2 describing the integer division.

b) Evaluate the following arithmetic and boolean expressions using the evaluation relations. You may use your rule defined in part a).

- $a = (10 + x \cdot y) - (z \text{ div } 3)$ with $\sigma(x) = 3,\ \sigma(y) = 4,\ \sigma(z) = 5$
- $b_1 = (true \vee false) \wedge \neg false$
- $b_2 = ((4 + x) = 7) \wedge (\neg(y > 5))$ with $\sigma(x) = 3,\ \sigma(y) = 7$

Exercise 3 (Structural Induction): (3 Points)

Consider the structure of lists of natural numbers l with the following inductive definition:

$$\frac{}{[] \in Lists(\mathbb{N})} \qquad \frac{l \in Lists(\mathbb{N})}{n :: l \in Lists(\mathbb{N})}$$

Where [] is the empty list, while the infix operator :: represents prefixing by a natural number. So, if n is a natural number and l is a list $n :: l$ is an extended list whose first element is n and remainder is the original list l.

We additionally define the following three operations on lists.

- $max(l)$ - the largest element in l (or 0 if l is empty)
- $sum(l)$ - the sum of the elements in l
- $len(l)$ - the length of the list l

Use structural induction to show the following property:

$$sum(l) \leq max(l) \cdot len(l)$$