

Exercise 1 (AM for While):

(1+1+1+1 Points)

- a) Write a program in the WHILE programming language computing the greatest common divisor of two positive numbers given by variables x and y .
- b) Translate the program into intermediate code.
- c) Give a run of the program, i.e. the sequence of program states for $x = 4, y = 2$.
- d) Show that it actually does compute the greatest common divisor using formal semantics.

Exercise 2 (AM: Repeat Until):

(1+1 Points)

- a) Extend the WHILE language of the lecture with the construct **repeat** c **until** b and specify the corresponding translation function.
- b) When modeling the **repeat** c **until** b via the similar construct $c; \text{while } \neg b \text{ do } c$; containing a while-loop in a straightforward way, the body c will be translated twice. Can you think of a way to avoid this double translation of c ?

Exercise 3 (Compiler Correctness):

(5 Points)

Provide the second/missing proof step for theorem of lecture 17, i.e. show that the following lemma holds:

Lemma 1 For every $c \in \text{Cmd}$, $\sigma, \sigma' \in \Sigma$ and $e \in \text{Stk}$, $\langle \mathfrak{T}_c \llbracket c \rrbracket, \epsilon, \sigma \rangle \triangleright^* \langle \epsilon, e, \sigma' \rangle$ implies $\langle c, \sigma \rangle \rightarrow \sigma'$ and $e = \epsilon$.

You may use all theorems and lemmata presented in the lecture (except from the one to proof). Additionally you may find the following lemma useful.

Lemma 2 (Decomposition Lemma) If $\langle c_1 : c_2, e, s \rangle \triangleright^k \langle \epsilon, e'', \sigma'' \rangle$, then there exists a configuration $\langle \epsilon, e', \sigma' \rangle$ and natural numbers k_1, k_2 with $k_1 + k_2 = k$ such that $\langle c_1, e, \sigma \rangle \triangleright^{k_1} \langle \epsilon, e', \sigma' \rangle$ and $\langle c_2, e', \sigma' \rangle \triangleright^{k_2} \langle \epsilon, e'', \sigma'' \rangle$.