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Hinweise:

- Note that content for exercises 3 to 5 is discussed in lecture 3. If you would like to try solving the exercises before, you may find the following execution relation helpful:

$$(skip)\ \frac{}{\langle skip, \sigma\rangle \to \sigma}$$

$$(seq)\ \frac{\langle c_1, \sigma\rangle \to \sigma' \qquad \langle c_1, \sigma\rangle \to \sigma''}{\langle c_1; c_2, \sigma\rangle \to \sigma''}$$

$$(asgn)\ \frac{\langle a, \sigma\rangle \to z}{\langle x := a, \sigma\rangle \to \sigma[x \mapsto z]}$$

$$(wh-f)\ \frac{\langle b, \sigma\rangle \to false}{\langle \textbf{while}\ b\ \textbf{do}\ c, \sigma\rangle \to \sigma}$$

$$(wh-t)\ \frac{\langle b, \sigma\rangle \to true \qquad \langle c, \sigma\rangle \to \sigma' \qquad \langle \textbf{while}\ b\ \textbf{do}\ c, \sigma'\rangle \to \sigma''}{\langle \textbf{while}\ b\ \textbf{do}\ c, \sigma\rangle \to \sigma''}$$

Exercise 1 (Single-Step Semantics): (3 Points)

In the lecture we have defined a so–called *bigstep semantics* for expressions, i.e., a relation $\to \subseteq (AExp \cup BExp) \times \Sigma \times (\mathbb{Z} \cup \mathbb{B})$ which yields the value of an expression within one step: $\langle (3+3) * (9-2), \sigma\rangle \to 42$. (Thus the intermediate results of the computation are "hidden" in the derivation tree.)
Alternatively it is possible to explicitly represent the intermediate steps by defining a *single–step semantics*: $\langle (3+3) * (9-2), \sigma\rangle \to_1^a \langle 6 * (9-2), \sigma\rangle \to_1^a \langle 6 * 7, \sigma\rangle \to_1^a \langle 42, \sigma\rangle \to_1^a 42$. Give a complete specification of the single–step relation

1. $\to_1^a \subseteq (AExp \times \Sigma) \times (AExp \times \Sigma \cup \mathbb{Z})$ for arithmetic expressions and
2. $\to_1^b \subseteq (BExp \times \Sigma) \times (BExp \times \Sigma \cup \mathbb{B})$ for Boolean expressions.

Exercise 2 (Side Effects): (3 Points)

In our WHILE language, the evaluation of (arithmetic) expressions has no *side effects*—it does not change the state. If we were to model side effects it would be natural to consider an evaluation relation of the form

$$\langle a, \sigma\rangle \to \langle z, \sigma'\rangle$$

where σ' ist the state that results from the evaluation of a in the original state σ. To introduce side effects in WHILE, extend the arithmetic expressions by a construct

$$c\ \textbf{resultis}\ a$$

where $c \in \textbf{Cmd}$ and $a \in \textbf{AExp}$. To evaluate such an expression, c is first executed and then a is evaluated in the new state. Formalize this idea by giving it an operational semantics.

Exercise 3 (Execution Relation): (2 Points)

Evaluate the following statement using the execution relations.

$$\text{while } x > 0 \text{ do } x := x - 1 \quad \text{with } \sigma(x) = 2$$

Exercise 4 (Non-Total Execution Relation): (3 Points)

Show that the following proposition holds:

For all states σ, σ', $\langle \textbf{while true do skip}, \sigma \rangle \nrightarrow \sigma'$.