

Exercise 1 (Unwinding of Loops):

(1+2+3 Points)

- a) Extend the WHILE language by a loop construct of the form

repeat c **until** b

and define its execution relation \rightarrow without (explicitly) using the **while** statement.

- b) Establish the following semantic equivalence:

repeat c **until** $b \quad \sim \quad c; \text{ if } b \text{ then skip else (repeat } c \text{ until } b).$

- c) Establish the following semantic equivalence:

repeat c **until** $b \quad \sim \quad c; \text{ while } \neg b \text{ do } c.$

Exercise 2 (Computing The Factorial):

(4 Points)

Show that the statement $c \in \text{Cmd}$ given by

$y := 1; \text{ while } \neg(x = 1) \text{ do } (y := y * x; x := x - 1)$

computes the factorial, i.e., that its operational semantics satisfies the following condition:

$$\mathfrak{D}(\sigma)(y) = (\sigma(x))!$$

for every $\sigma \in \Sigma$ with $\sigma(x) \geq 1$.