

Exercise 1 (Denotational Evaluation):

(2 Points)

Evaluate the following program by means of the denotational semantic functional.

```

a := 0;
if (x < 0) then
  a := -1;
else
  if (x > 0) then
    a := 1;
  else
    x := a + 1;

```

Exercise 2 (Strict Composition):

(2+2+2 Points)

Compute the following compositions of f , g and h , where

$f, g, h: \Sigma \dashrightarrow \Sigma$

$\sigma_x \in \Sigma$

$\circ: (\Sigma \dashrightarrow \Sigma) \times (\Sigma \dashrightarrow \Sigma) \rightarrow (\Sigma \dashrightarrow \Sigma)$

$$f(\sigma_x) := \begin{cases} \sigma'_x & \text{if } \sigma(x) = 1 \vee 0 \\ \text{undef.} & \text{otherwise} \end{cases}$$

$$g(\sigma_x) := \begin{cases} \sigma'_x & \text{if } \sigma(x) = 1 \\ \sigma_x & \text{otherwise} \end{cases}$$

$$h(\sigma_x) := \begin{cases} \sigma_1 & \text{if } g(\sigma_x) = \sigma_x \\ \sigma_0 & \text{otherwise} \end{cases}$$

a) $(f \circ g \circ h)(\sigma_x)$ **b)** $(g \circ f \circ h)(\sigma_x)$ **c)** $(g \circ h \circ f)(\sigma_x)$

Exercise 3 (Denotational Evaluation of While):

(4 Points)

Assume we define the denotational semantics of the while-statement as follows:

$$\mathcal{C}[\text{while } b \text{ do } c]\sigma := \begin{cases} \mathcal{C}[c; \text{while } b \text{ do } c], & \text{if } \mathcal{B}[b]\sigma = \text{true} \\ \sigma, & \text{otherwise} \end{cases}$$

Consider the while-program c :

```

g := 0;
while (x >= y) do
  g := g + 1;
  x := x - y;

```

Show that the following statement about c holds (consider $\sigma_{x,y,g}$):

$\mathcal{D}[c]\sigma_{m,n,o} = \sigma_{r,n,q}$ with $m = q * n + r$ and $r < n$ where $m, n \in \mathbb{N}$, $n > 0$.