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Exercise 1 (Monotonicity): (3 Points)

Which of the following functionals of type $(\Sigma \dashrightarrow \Sigma) \to (\Sigma \dashrightarrow \Sigma)$ are monotonic with respect to the partial order $\sqsubseteq$ given by graph inclusion and why?

1. $\Phi_1(f) = f$
2. $\Phi_2(f) = \begin{cases} g_1 & \text{if } f = g_2 \\ g_2 & \text{otherwise} \end{cases}$ (where $g_1, g_2 : \Sigma \dashrightarrow \Sigma$ with $g_1 \neq g_2$)
3. $\Phi_3(f)(\sigma) = \begin{cases} f(\sigma) & \text{if } \sigma(x) \neq 0 \\ \sigma & \text{otherwise} \end{cases}$

Exercise 2 (Equivalence of Operational and Denotational Semantics): (4 Points)

Show that the operational and the denotational semantics of boolean expressions coincide, i.e. prove the following result.
For every $b \in \mathbf{BExp}$, $\sigma \in \Sigma$, and $t \in \mathbb{B}$:

$$\langle b, \sigma \rangle \to t \quad \text{iff} \quad \mathfrak{B}[\![b]\!](\sigma) = t.$$

You may assume we already know that the operational and denotational semantics of arithmetic expressions coincide, thus fulfilling:
For every $a \in \mathbf{AExp}$, $\sigma \in \Sigma$, and $z \in \mathbb{Z}$:

$$\langle a, \sigma \rangle \to z \quad \text{iff} \quad \mathfrak{A}[\![a]\!](\sigma) = z.$$

Exercise 3 (Denotational Semantics of While): (2+1 Points)

Consider the following fragment of the factorial program:

$$\textbf{while } \neg(x = 1) \textbf{ do } (y := y * x;\ x := x - 1).$$

1. Determine the corresponding functional $\Phi : (\Sigma \dashrightarrow \Sigma) \to (\Sigma \dashrightarrow \Sigma)$.
2. Give at least two fixpoints of Φ.