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Exercise 1 (Continuity): (1+2 Points)

Show that the following propositions hold:

1. The identity function id_D on a cpo D is continuous.

2. Let $f : D \to E$ and $g : E \to F$ be continuous functions on cpos D, E, F. Then their composition $g \circ f : D \to F$ is continuous.

Exercise 2 (Fixpoint Approximation): (4 Points)

Investigate

$$\mathfrak{C}[\![z := 0; \textbf{while } x > 0 \textbf{ do } (x := x - 1; z := z + y)]\!]$$

in analogy to the factorial example 7.9 given in lecture 7.

Exercise 3 (Three-Valued Denotation Semantics): (1+1+2 Points)

Define a three-valued denotational semantics for the *WHILE* language as follows:

1. Assume that at the beginning of a programm evaluation, all variables have unknown values. To model this, extend the variable domain by $\bot$, and let $\sigma_\bot$ with $\sigma(x) = \bot$ for all $x \in \textbf{Var}$ be the initial *state* of all programs. Define $\mathfrak{A}[\![\cdot]\!]$ in analogy to Definition 5.1 and evaluate $3 + x$ and $0 * x$ for $\sigma_\bot$.

2. In addition to true and false, a third truth-value ? is needed to express uncertainty about the result of a boolean expression, i.e. $x > 0$ may hold or not, depending on how x is initialized, and thus it should evaluate to ?. Define $\mathfrak{B}[\![\cdot]\!]$ in analogy to Definition 5.2 and evaluate $\neg(x = y) \wedge false$ for $\sigma_\bot$.

3. Define **cond** such that common evaluation results are preserved in case of an indefinite evaluation of the boolean expression. Evaluate $\textbf{cond}(?, \mathfrak{C}[\![x := 2; y := 3; z := x + y]\!], \mathfrak{C}[\![x := 3; y := 2; z := x + y]\!])$ for $\sigma_\bot$.