AOR Priv.-Doz. Dr. Thomas Noll

Christina Jansen, Sabrina von Styp

Exercise 1 (Interpretation of Logical Variables): **(2+2 Points)**

a) Prove by structural induction on expressions $a \in \textbf{LExp}$ and $n \in \mathbb{N}$ that

$$\mathfrak{L}[\![a]\!]I[n/i]\sigma = \mathfrak{L}[\![a[n/i]]\!]I\sigma.$$

b) By using the fact above, for any $A \in \textbf{Assn}$, prove

$$\sigma \models^I \forall i.A \text{ iff } \sigma \models^I A[n/i] \text{ for all } n \in \mathbb{N}.$$

Exercise 2 (Axiomatic Equivalence): **(4 Points)**

Establish the following axiomatic equivalence:

$$\textbf{for } x := a_1 \textbf{ to } a_2 \textbf{ do } c \quad \equiv \quad x := a_1;\ \textbf{while } x \leq a_2 \textbf{ do } (c;\ x := x + 1)$$

where the semantics of the for-statement are given as

$$\frac{\{A \wedge x \leq a_2\}\ c\ \{A[x \mapsto x + 1]\}}{\{A[x \mapsto a_1]\}\ \textbf{for } x := a_1 \textbf{ to } a_2 \textbf{ do } c\ \{A \wedge x > a_2\}}$$

without using the equivalence of axiomatic and operational/denotational semantics.

Exercise 3 (Assertion Language): **(1+2 Points)**

We extend the set of boolean assertions *Assn* by a further rule $A ::= \exists i.A \in Assn$.

a) Give semantics for the new assertion by expressing it by means of already given assertions or extend the axiomatic functional accordingly.

b) Using your semantics from a), show that the following proposition holds:

$$\sigma \models^I \exists i.A \Leftrightarrow \sigma \models^I A[i \rightarrow n] \text{ for some } n \in \mathbb{N}$$