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Exercise 1 (Total Correctness): (3 Points)

Prove the total correctness of the following program:

$$\{A\}\ z := 0; c\ \{\Downarrow B\}$$

where

$$\begin{aligned} A &:= (x > 0 \wedge x = i \wedge y > 0) \\ c &:= \textbf{while } y \leqslant x \textbf{ do } (z := z + 1; x := x - y) \\ B &:= (z = i \textbf{ div } y) \end{aligned}$$

Exercise 2 (Questions about Axiomatic Semantics): (4 Points)

To conclude the section about axiomatic semantics, answer the following questions.

- What is the idea of axiomatic semantics? Which conclusions about program behaviour can be drawn by using them?
- Describe the concept of assertions and partial correctness properties.
- What do you know about soundness and completeness of axiomatic semantics?
- Briefly sketch the steps needed to prove total correctness of a program fragment. Which statements about termination of a program fragment can be proven by means of partial correctness properties?

Exercise 3 (Axiomatic Equivalence): (3 Points)

Consider the axiomatic equivalence of two statements defined in Definition 12.1 in the lecture. Based on this definition show that the following proposition holds:

Proposition 1 $\forall A, B \in Assn : c_1 \approx c_2 \Leftrightarrow (\models \{A\}\ c_1\ \{\Downarrow B\} \Leftrightarrow \models \{A\}\ c_2\ \{\Downarrow B\})$.

Exercise 4 (Operational Semantics Procedure Blocks): (2 Points)

Consider the following example:

$c_0 \equiv$
```
begin
  var x; var y;
  proc P is y := x;
  x := 1;
  begin
    var x;
    x := 2;
    call P
  end
end
```

Choose initial variable environment ρ_0 such that with static scope semantics, for all π_0 and σ_0:

$$\rho_0, \pi_0 \vdash \langle c_0, \sigma_0 \rangle \to \sigma_0[0 \mapsto 1, 1 \mapsto 1, 2 \mapsto 2]$$

Give a formal proof, using the operational semantics in Definition 12.4.