

Semantics and Verification of Software

Lecture 14: Extension by Blocks and Procedures I (Operational Semantics)

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- 1 Extension by Blocks and Procedures
- 2 Operational Semantics of Blocks and Procedures

- Extension of WHILE by **blocks** with (local) **variables** and (recursive) **procedures**
- Simple memory model ($\Sigma := \{\sigma \mid \sigma : \text{Var} \rightarrow \mathbb{Z}\}$) not sufficient anymore as variables can occur in several **instances**

⇒ Involves new semantic concepts:

- variable und procedure **environments**
- **locations** (memory addresses) and **stores** (memory states)

- Important: **scope** of variable and procedure identifiers
 - static scoping: scope of identifier = **declaration environment**
(here)
 - dynamic scoping: scope of identifier = **calling environment**
(old Algol/Lisp dialects)

Static and Dynamic Scoping

Example 14.1

```
begin
  var x; var y;
  proc P is y := x;
  x := 1;
  begin
    var x;
    x := 2;
    call P
  end
end
```

static scoping $\Rightarrow y = 1$
dynamic scoping $\Rightarrow y = 2$

Syntactic categories:

Category	Domain	Meta variable
Procedure identifiers	$PVar = \{P, Q, \dots\}$	P
Procedure declarations	$PDec$	p
Variable declarations	$VDec$	v
Commands (statements)	Cmd	c

Context-free grammar:

$$p ::= \text{proc } P \text{ is } c; p \mid \varepsilon \in PDec$$
$$v ::= \text{var } x; v \mid \varepsilon \in VDec$$
$$c ::= \text{skip} \mid x := a \mid c_1; c_2 \mid \text{if } b \text{ then } c_1 \text{ else } c_2 \mid \text{while } b \text{ do } c \mid \text{call } P \mid \text{begin } v \mid p \mid c \mid \text{end} \in Cmd$$

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- So far: **states** $\Sigma = \{\sigma \mid \sigma : \text{Var} \rightarrow \mathbb{Z}\}$
- Now: explicit control over all (nested) **instances** of a variable:
 - **variable environments** $VEnv := \{\rho \mid \rho : \text{Var} \rightarrow \text{Loc}\}$
 - **locations** $\text{Loc} := \mathbb{N}$
 - **stores** $\text{Sto} := \{\sigma \mid \sigma : \text{Loc} \rightarrow \mathbb{Z}\}$
(**partial** function to maintain allocation information)

⇒ **Two-level access** to a variable $x \in \text{Var}$:

- ① determine current memory location of x :

$$l := \rho(x)$$

- ② reading/writing access to σ at position l

- Thus: previous **state** information represented as $\sigma \circ \rho$

- Effect of procedure call determined by its body and variable and procedure environment of its declaration:

$$PEnv := \{\pi \mid \pi : PVar \rightarrow Cmd \times VEnv \times PEnv\}$$

denotes the set of procedure environments

- Effect of declaration: update of environment (and store)

$$upd_v[[\cdot]] : VDec \times VEnv \times Sto \rightarrow VEnv \times Sto$$

$$\begin{aligned} upd_v[[\text{var } x; v]](\rho, \sigma) &:= upd_v[[v]](\rho[x \mapsto l_x], \sigma[l_x \mapsto 0]) \\ upd_v[[\varepsilon]](\rho, \sigma) &:= (\rho, \sigma) \end{aligned}$$

$$upd_p[[\cdot]] : PDec \times VEnv \times PEnv \rightarrow PEnv$$

$$\begin{aligned} upd_p[[\text{proc } P \text{ is } c; p]](\rho, \pi) &:= upd_p[[p]](\rho, \pi[P \mapsto (c, \rho, \pi)]) \\ upd_p[[\varepsilon]](\rho, \pi) &:= \pi \end{aligned}$$

where $l_x := \min\{l \in Loc \mid \sigma(l) = \perp\}$

Execution Relation I

Definition 14.2 (Execution relation)

For $c \in Cmd$, $\sigma, \sigma' \in Sto$, $\rho \in VEnv$, and $\pi \in PEnv$, the **execution relation** $(\rho, \pi) \vdash \langle c, \sigma \rangle \rightarrow \sigma'$ ("in environment (ρ, π) , statement c transforms store σ into σ' ") is defined by the following rules:

$$\frac{(\text{skip})}{(\rho, \pi) \vdash \langle \text{skip}, \sigma \rangle \rightarrow \sigma}$$
$$\frac{(\text{asgn}) \quad \langle a, \sigma \circ \rho \rangle \rightarrow z}{(\rho, \pi) \vdash \langle x := a, \sigma \rangle \rightarrow \sigma[\rho(x) \mapsto z]}$$
$$\frac{(\rho, \pi) \vdash \langle c_1, \sigma \rangle \rightarrow \sigma' \quad (\rho, \pi) \vdash \langle c_2, \sigma' \rangle \rightarrow \sigma''}{(\rho, \pi) \vdash \langle c_1 ; c_2, \sigma \rangle \rightarrow \sigma''}$$
$$\frac{(\text{if-t}) \quad \langle b, \sigma \circ \rho \rangle \rightarrow \text{true} \quad (\rho, \pi) \vdash \langle c_1, \sigma \rangle \rightarrow \sigma'}{(\rho, \pi) \vdash \langle \text{if } b \text{ then } c_1 \text{ else } c_2, \sigma \rangle \rightarrow \sigma'}$$
$$\frac{(\text{if-f}) \quad \langle b, \sigma \circ \rho \rangle \rightarrow \text{false} \quad (\rho, \pi) \vdash \langle c_2, \sigma \rangle \rightarrow \sigma'}{(\rho, \pi) \vdash \langle \text{if } b \text{ then } c_1 \text{ else } c_2, \sigma \rangle \rightarrow \sigma'}$$

Definition 14.2 (Execution relation; continued)

$$(\text{wh-f}) \frac{\langle b, \sigma \circ \rho \rangle \rightarrow \text{false}}{(\rho, \pi) \vdash \langle \text{while } b \text{ do } c, \sigma \rangle \rightarrow \sigma}$$

$$(\text{wh-t}) \frac{\langle b, \sigma \circ \rho \rangle \rightarrow \text{true} \quad (\rho, \pi) \vdash \langle c, \sigma \rangle \rightarrow \sigma' \quad (\rho, \pi) \vdash \langle \text{while } b \text{ do } c, \sigma' \rangle \rightarrow \sigma''}{(\rho, \pi) \vdash \langle \text{while } b \text{ do } c, \sigma \rangle \rightarrow \sigma''}$$

$$(\text{call}) \frac{(\rho', \pi'[P \mapsto (c, \rho', \pi')]) \vdash \langle c, \sigma \rangle \rightarrow \sigma'}{(\rho, \pi) \vdash \langle \text{call } P, \sigma \rangle \rightarrow \sigma'} \quad \text{if } \pi(P) = (c, \rho', \pi')$$

$$(\text{block}) \frac{\text{upd}_v[v](\rho, \sigma) = (\rho', \sigma') \quad (\rho', \text{upd}_p[p](\rho', \pi)) \vdash \langle c, \sigma' \rangle \rightarrow \sigma''}{(\rho, \pi) \vdash \langle \text{begin } v \ p \ c \ \text{end}, \sigma \rangle \rightarrow \sigma''}$$

Remarks about rules (call) and (block):

- **Static scoping** is modelled in (call) by using the environments ρ' and π' (as determined in (block)) from the **declaration** site of procedure P (and not ρ and π from the **calling** site)
- In (call), the procedure environment associated with procedure P is extended by a P -entry to handle **recursive calls** of P :

$$\pi'[P \mapsto (c, \rho', \pi')]$$

Example 14.3

```

c = begin
    var x; var y; } v
    proc F is
        begin
            var z;
            z := x;
            if z=1 then skip
                else x := x-1;
                call F;
                y := z * y } c2 } c1 } cF } p
            end
            x := 2; y := 1; call F } c0
        end
    
```

Let $\sigma_\emptyset(l) = \rho_\emptyset(x) = \pi_\emptyset(P) = \perp$ for all $l \in Loc, x \in Var, P \in PVar$

Notation: $\sigma_{ijkl} \Leftrightarrow \sigma(0) = i, \sigma(1) = j, \sigma(2) = k, \sigma(3) = l$

Derivation tree for $(\rho_\emptyset, \pi_\emptyset) \vdash \langle c, \sigma_\emptyset \rangle \rightarrow \sigma_{1221}$: on the board