apl. Prof. Dr. Thomas Noll

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E-mail your solution to kvdpol at cs.rwth-aachen.de *before* April 22, 10:00 CET.

Exercise 1 (Programming Language WHILE): (2 + 1 Points)

a) Assume that in WHILE programming language an additional operator *div* is given to compute the quotient of two numbers. Write a program using the WHILE programming language that calculates z := gcd (x ,y) with x,y,z $\in \mathbb{Z}$, where z is the greatest common divisor of x and y.

b) Draw the flow diagram for the program from part a).

Exercise 2 (Evaluation Relations): (1 + 3 Points)

a) Declare a rule for the arithmetic expression gcd (a_1, a_2) describing the greatest common divisor of integers.

b) Evaluate the following arithmetic and boolean expressions using the evaluation relations. You may use your rule defined in part a).

- $a = \text{gcd}\ (z, 3) + (x * 5 + y)$ with $\sigma(x) = 2,\ \sigma(y) = -10,\ \sigma(z) = 9$
- $b_1 = (true \wedge \neg false) \vee false$
- $b_2 = (3 > z) \vee (\neg(x + 2 > y))$ with $\sigma(x) = 3,\ \sigma(y) = 5,\ \sigma(z) = 1$

Exercise 3 (Structural Induction): (3 Points)

Consider $\mathcal{T}$, the structure of binary trees, with the following inductive definition:

$$\frac{}{\text{leaf} \in \mathcal{T}} \qquad \frac{t_1 \in \mathcal{T} \qquad t_2 \in \mathcal{T}}{\text{node}(t_1, t_2) \in \mathcal{T}}$$

We additionally define the following two operations on trees.

- $\text{height}(t)$ - the number of nodes/leaves on the longest path from root to leaf:

$$\text{height}(\text{leaf}) = 1$$
$$\text{height}(\text{node}(t_1, t_2)) = 1 + \max(\text{height}(t_1), \text{height}(t_2))$$

- $\text{size}(t)$ - the total number of nodes/leaves in the tree

For example, consider the tree depicted in Figure 1. Its size is 5 and its height is 3.
Use structural induction to show the following property for all $t \in \mathcal{T}$:

$$\text{size}(t) \leq 2^{\text{height}(t)} - 1$$

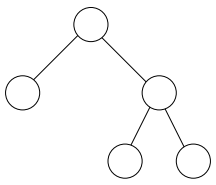

Abbildung 1: The tree node(leaf, node(leaf, leaf)).