

### Exercise 1 (Extended CCS):

**(6 Points)**

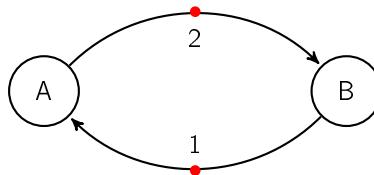
Many extended CCS languages are widely used in modelling different systems. Now we focus on an extended CCS which allows

- processes to have parameterized value variables (here we restrict them to be of type integer), and
- such values (= evaluation of variables) can be passed to other processes during synchronization,
- to add a **if** bexp **then** P **else** Q construct such that the process can make decision based on the current values of its variables.

An example of one-place buffer which only store an integer bigger than 10 can be defined by using such extended CCS language as

$$\begin{aligned}
 B &= \text{input}(x).B(x), \\
 B(x) &= \text{if } x > 10 \text{ then } \overline{\text{output}(x)}.B \text{ else } \text{discard}.B.
 \end{aligned}$$

- This extension of CCS introduces a *state*, a valuation of variables, in the semantics. What is the new type of the execution relation?
- Modify the rules in Definition 18.3 to define the semantics of this extended CCS language.
- Consider the following system. Two processes are connected by two directed channels which can store tokens. As soon as the channel A to B has more than 2 tokens, process B can execute and produce 1 token to the channel from B to A, after consuming 2 tokens from the channel A to B. In the same way, as soon as the channel A to B has more than 1 token, process A can execute and produce 2 tokens to the channel from A to B, after consuming 1 token from the channel B to A. Please model this system by using extended CCS.



- Now the channels are initialized with 2 tokens in channel A to B and 1 token in channel B to A. Initialize your system and give the corresponding LTS based on your definition.
- Now we would like to use a model of a clock to measure the time passing (one tick represents one second). We define it as follows:

$$\text{Clock} = \overline{\text{tick}}.\text{Clock}$$

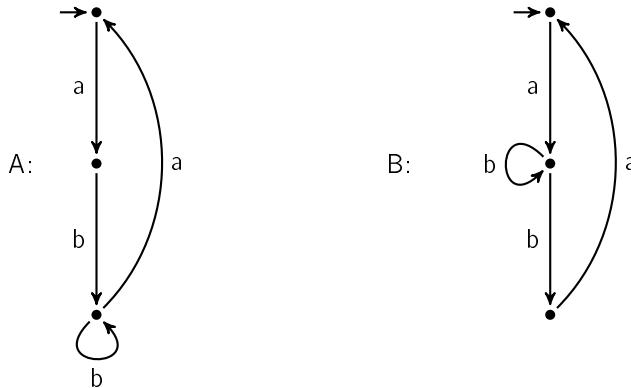
Now we assume A's execution takes one second and B's execution takes two seconds, and synchronization between processes happens immediately. Why can this not be modeled in the extended CCS?

- Change the extended CCS semantics to fix this problem. Add the clock to the model such that the processes correctly synchronize.

**Exercise 2 (Bisimulation in CCS):**

**(1+1+2 Points)**

Assume following LTSs according to two CCS processes are given:



- a) Are following two LTSs  $A$  and  $B$  trace equivalent?
- b) Are they bisimilar? Justify your answers.
- c) Give an LTS as a triple  $(S, Act, \rightarrow)$  where  $S$  is a nonempty set of state (i.e. processes in CCS),  $Act$  is a set of actions, and  $\rightarrow \subseteq S \times Act \times S$  is a set transitions. A so-called Hennessy-Milner logic (HML) defines a set of formulae  $\mathcal{M}$  over  $Act$  to describe the properties in LTS as:

$$F, G ::= \text{true} \mid \text{false} \mid F \wedge G \mid F \vee G \mid \langle a \rangle F \mid [a]F \quad (\text{where } a \in Act)$$

Let  $F$  be a formula in  $\mathcal{M}$ , a state  $p \in S$  satisfies  $F$  (denote as  $p \models F$ ) is defined as follows:

- $p \models \text{true}$  iff  $p \in S$ ,
- $p \models \text{false}$  iff  $p \in \emptyset$ ,
- $p \models \langle a \rangle$  iff  $\exists q \models F. p \xrightarrow{a} q$ ,
- $p \models [a]$  iff  $\forall q \in S. p \xrightarrow{a} q \Rightarrow q \models F$ .

Give a formula  $F$  in HML which can distinguish these two LTSs.