apl. Prof. Dr. Thomas Noll

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Please hand in your solutions in groups of two or three students. Your solutions will not be corrected otherwise.

Exercise 1 (CCPOs and Continuity): (1+1+1 Points)

Show that the following propositions hold:

1. If $(D_1, \leq)$ and $(D_2, \leq)$ are ccpos, then the cartesian product $(D_1 \times D_2, \dot{\leq})$ is a ccpo, where $(x, y)\dot{\leq}(x', y')$ iff. $x \leq x'$ and $y \leq y'$.
2. The identity function id_D on a cpo D is continuous.
3. Let $f : D \to E$ and $g : E \to F$ be continuous functions on cpos D, E, F. Then their composition $g \circ f : D \to F$ is continuous.

Exercise 2 (Operational and denotational equivalence): (3 Points)

We want to prove that the operational semantics and denotational semantics of the *WHILE* language are equivalent. Assume that we have already shown that the operational and the denotational semantics of boolean expressions and arithmetic expressions in the *WHILE* language coincide. Show that the operational and the denotational semantics of commands coincide, i.e. for every $c \in Cmd$:

$$\mathfrak{O}[\![c]\!] = \mathfrak{C}[\![c]\!] .$$

Exercise 3 (Three-Valued Denotation Semantics): (1+1+2 Points)

Define a three-valued denotational semantics for the *WHILE* language as follows:

1. Assume that at the beginning of a program evaluation, all variables have unknown values. To model this, extend the variable domain by $\perp$, and let $\sigma_\perp$ with $\sigma(x) = \perp$ for all $x \in \mathbf{Var}$ be the initial *state* of all programs. Define $\mathfrak{A}[\![\cdot]\!]$ in analogy to Definition 5.1 and evaluate $x - x$ and $0 * x$ for $\sigma_\perp$.
2. In addition to true and false, a third truth-value ? is needed to express uncertainty about the result of a boolean expression, i.e. $x > 0$ may hold or not, depending on how x is initialized, and thus it should evaluate to ?. Define $\mathfrak{B}[\![\cdot]\!]$ in analogy to Definition 5.2 and evaluate $x < y \vee true$ for $\sigma_\perp$.
3. Define **cond** such that common evaluation results are preserved in case of an indefinite evaluation of the boolean expression. Evaluate $\mathbf{cond}(?, \mathfrak{C}[\![x := 2; y := 6; z := x * y]\!], \mathfrak{C}[\![x := 4; y := 3; z := x * y]\!])$ for $\sigma_\perp$.