

Semantics and Verification of Software

Lecture 18: Nondeterminism and Parallelism III (Calculus of Communicating Systems)

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1 Calculus of Communicating Systems

2 Semantics of CCS

History:

- Robin Milner: *A Calculus of Communicating Systems*
LNCS 92, Springer, 1980
- Robin Milner: *Communication and Concurrency*
Prentice-Hall, 1989
- Robin Milner: *Communicating and Mobile Systems: the π -calculus*
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Approach: describing parallelism on a **simple and abstract level**, using only a few basic primitives

- no explicit storage (variables)
- no explicit representation of values (numbers, Booleans, ...)

⇒ parallel system reduced to **communication potential**

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- Let Pid be a set of **process identifiers**.
- The set Prc of **process expressions** is defined by the following syntax:

$$\begin{array}{ll} P ::= \text{nil} & \text{(inaction)} \\ | \quad \alpha.P & \text{(prefixing)} \\ | \quad P_1 + P_2 & \text{(choice)} \\ | \quad P_1 \parallel P_2 & \text{(parallel composition)} \\ | \quad \text{new } a P & \text{(restriction)} \\ | \quad A(a_1, \dots, a_n) & \text{(process call)} \end{array}$$

where $\alpha \in Act$, $a, a_i \in N$, and $A \in Pid$.

Definition 18.1 (continued)

- A **(recursive) process definition** is an equation system of the form

$$(A_i(a_{i1}, \dots, a_{in_i}) = P_i \mid 1 \leq i \leq k)$$

where $k \geq 1$, $A_i \in \text{Pid}$ (pairwise different), $n_i \in \mathbb{N}$, $a_{ij} \in N$ (a_{i1}, \dots, a_{in_i} pairwise different), and $P_i \in \text{Prc}$ (with process identifiers from $\{A_1, \dots, A_k\}$).

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Notational Conventions:

- \bar{a} means a
- $A(a_1, \dots, a_n)$ sometimes written as $A(\vec{a})$, $A()$ as A
- prefixing and restriction binds stronger than composition, composition binds stronger than choice:

$$\text{new } a \, P + b.Q \parallel R \quad \text{means} \quad (\text{new } a \, P) + ((b.Q) \parallel R)$$

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Meaning of CCS Constructs

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- An action $a \in N$ ($\bar{a} \in \bar{N}$) is interpreted as an **input** (**output**, resp.) operation. Both are complementary: if executed in parallel (i.e., in $P_1 \parallel P_2$), they are merged into a τ -action.

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- The **restriction** $\text{new } a.P$ declares a as a local name which is only known within P .
- The behavior of a **process call** $A(a_1, \dots, a_n)$ is defined by the right-hand side of the corresponding equation where a_1, \dots, a_n replace the formal name parameters.

Example 18.2

(on the board)

- ➊ One-place buffer (see Example 17.1(1) for a CSP implementation)
- ➋ Two-place buffer
- ➌ Parallel specification of two-place buffer
(see Example 17.1(2) for a CSP implementation)

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Definition 18.3 (Semantics of CCS)

A process definition $(A_i(a_{i1}, \dots, a_{in_i}) = P_i \mid 1 \leq i \leq k)$ determines the **labeled transition system (LTS)** (Prc, Act, \rightarrow) whose transitions can be inferred from the following rules ($P, P', Q, Q' \in Prc$, $\alpha \in Act$, $\lambda \in N \cup \bar{N}$, $a, b \in N$, $A \in Pid$):

$$(Act) \frac{}{\alpha.P \xrightarrow{\alpha} P}$$

$$(Sum_1) \frac{P \xrightarrow{\alpha} P'}{P + Q \xrightarrow{\alpha} P'}$$

$$(Par_1) \frac{P \xrightarrow{\alpha} P'}{P \parallel Q \xrightarrow{\alpha} P' \parallel Q}$$

$$(New) \frac{P \xrightarrow{\alpha} P' \ (\alpha \notin \{a, \bar{a}\})}{\text{new } a \ P \xrightarrow{\alpha} \text{new } a \ P'}$$

$$(Com) \frac{P \xrightarrow{\lambda} P' \ Q \xrightarrow{\bar{\lambda}} Q'}{P \parallel Q \xrightarrow{\tau} P' \parallel Q'}$$

$$(Sum_2) \frac{Q \xrightarrow{\alpha} Q'}{P + Q \xrightarrow{\alpha} Q'}$$

$$(Par_2) \frac{Q \xrightarrow{\alpha} Q'}{P \parallel Q \xrightarrow{\alpha} P \parallel Q'}$$

$$(Call) \frac{P[\vec{a} \mapsto \vec{b}] \xrightarrow{\alpha} P'}{A(\vec{b}) \xrightarrow{\alpha} P'} \text{ if } A(\vec{a}) = P$$

(Here $P[\vec{a} \mapsto \vec{b}]$ denotes the replacement of every a_i by b_i in P .)

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- ② Sequential two-place buffer:

$$B_0(in, out) = in.B_1(in, out)$$

$$B_1(in, out) = \overline{out}.B_0(in, out) + in.B_2(in, out)$$

$$B_2(in, out) = \overline{out}.B_1(in, out)$$

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- ③ Parallel two-place buffer:

$$B_{\parallel}(in, out) = \text{new com } (B(in, com) \parallel B(com, out))$$

$$B(in, out) = in.\overline{out}.B(in, out)$$

Example 18.4 (continued)

Complete LTS of parallel two-place buffer ($=: LTS(B_{\parallel}(in, out))$):

