Semantics and Verification of Software

Lecture 19: Nondeterminism and Parallelism IV (Equivalence of CCS Processes)

Thomas Noll

Lehrstuhl für Informatik 2
(Software Modeling and Verification)

RWTH AACHEN UNIVERSITY

noll@cs.rwth-aachen.de

http://www-i2.informatik.rwth-aachen.de/i2/svsw13/

Summer Semester 2013

Online Registration for Seminars and Practical Courses (Praktika) in Winter Term 2013/14

Who?

Students of:
- Master Courses
- Bachelor Informatik (~~Pro~~Seminar!)

Where?

www.graphics.rwth-aachen.de/apse

When?

05.07.2013 - 17.07.2013

Outline

Syntax of CCS I

Definition 19.1 (Syntax of CCS)

- Let N be a set of (action) names.
- $\overline{N} := \{\overline{a} \mid a \in N\}$ denotes the set of co-names.
- $Act := N \cup \overline{N} \cup \{\tau\}$ is the set of actions where τ denotes the silent (or: unobservable) action.
- Let Pid be a set of process identifiers.
- The set Prc of process expressions is defined by the following syntax:

$$
\begin{array}{rll}
P ::= & \text{nil} & \text{(inaction)} \\
\mid & \alpha.P & \text{(prefixing)} \\
\mid & P_1 + P_2 & \text{(choice)} \\
\mid & P_1 \parallel P_2 & \text{(parallel composition)} \\
\mid & \text{new } a\, P & \text{(restriction)} \\
\mid & A(a_1, \ldots, a_n) & \text{(process call)}
\end{array}
$$

where $\alpha \in Act$, $a, a_i \in N$, and $A \in Pid$.

Syntax of CCS II

Definition 19.1 (continued)

- A (recursive) process definition is an equation system of the form

$$(A_i(a_{i1}, \ldots, a_{in_i}) = P_i \mid 1 \leq i \leq k)$$

where $k \geq 1$, $A_i \in Pid$ (pairwise different), $n_i \in \mathbb{N}$, $a_{ij} \in N$ ($a_{i1}, \ldots, a_{in_i}$ pairwise different), and $P_i \in Prc$ (with process identifiers from $\{A_1, \ldots, A_k\}$).

Notational Conventions:

- $\overline{\overline{a}}$ means a
- $A(a_1, \ldots, a_n)$ sometimes written as $A(\vec{a})$, $A()$ as A
- prefixing and restriction binds stronger than composition, composition binds stronger than choice:

$$\text{new } a\, P + b.Q \parallel R \quad \text{means} \quad (\text{new } a\, P) + ((b.Q) \parallel R)$$

Definition 19.2 (Semantics of CCS)

A process definition $(A_i(a_{i1}, \ldots, a_{in_i}) = P_i \mid 1 \leq i \leq k)$ determines the labeled transition system (LTS) $(Prc, Act, \longrightarrow)$ whose transitions can be inferred from the following rules ($P, P', Q, Q' \in Prc$, $\alpha \in Act$, $\lambda \in N \cup \overline{N}$, $a, b \in N$, $A \in Pid$):

$$(\text{Act})\frac{}{\alpha.P \xrightarrow{\alpha} P} \qquad (\text{Com})\frac{P \xrightarrow{\lambda} P' \;\; Q \xrightarrow{\overline{\lambda}} Q'}{P \parallel Q \xrightarrow{\tau} P' \parallel Q'}$$

$$(\text{Sum}_1)\frac{P \xrightarrow{\alpha} P'}{P + Q \xrightarrow{\alpha} P'} \qquad (\text{Sum}_2)\frac{Q \xrightarrow{\alpha} Q'}{P + Q \xrightarrow{\alpha} Q'}$$

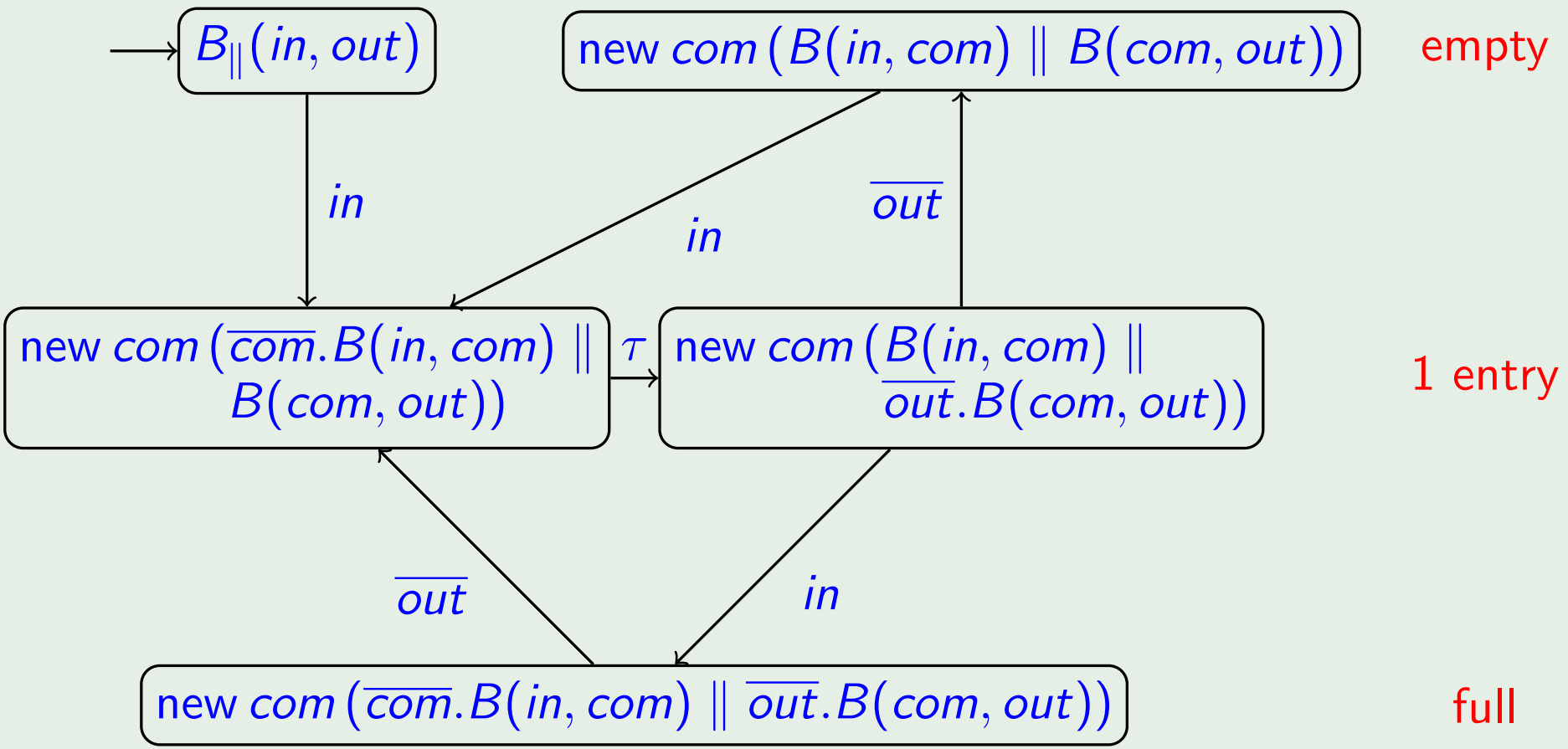
$$(\text{Par}_1)\frac{P \xrightarrow{\alpha} P'}{P \parallel Q \xrightarrow{\alpha} P' \parallel Q} \qquad (\text{Par}_2)\frac{Q \xrightarrow{\alpha} Q'}{P \parallel Q \xrightarrow{\alpha} P \parallel Q'}$$

$$(\text{New})\frac{P \xrightarrow{\alpha} P' \;\; (\alpha \notin \{a, \overline{a}\})}{\text{new } a\, P \xrightarrow{\alpha} \text{new } a\, P'} \qquad (\text{Call})\frac{P[\vec{a} \mapsto \vec{b}] \xrightarrow{\alpha} P'}{A(\vec{b}) \xrightarrow{\alpha} P'} \text{ if } A(\vec{a}) = P$$

(Here $P[\vec{a} \mapsto \vec{b}]$ denotes the replacement of every a_i by b_i in P.)

Example 19.3

Complete LTS of parallel two-place buffer (=: $LTS(B_{\|}(in, out))$):



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- **First idea:** define (for $P, Q \in Prc$)

  P, Q are called LTS equivalent if $LTS(P) = LTS(Q)$
- **But:** yields too many distinctions

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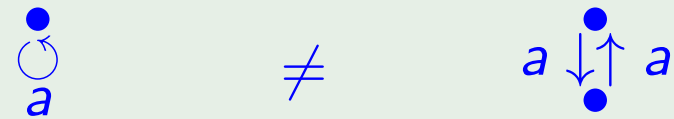
LTS: 

although both processes can (only) execute infinitely many a-actions, and should therefore be considered equivalent

Trace Equivalence I

Second idea: reduce process to its action sequences

Definition 19.2 (Trace language)

For every $P \in Prc$, let

$$Tr(P) := \{w \in Act^* \mid \text{ex. } P' \in Prc \text{ such that } P \xrightarrow{w} P'\}$$

be the trace language of P
(where $\xrightarrow{w} := \xrightarrow{a_1} \circ \ldots \circ \xrightarrow{a_1}$ for $w = a_1 \ldots a_n$).

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$P, Q \in Prc$ are called trace equivalent if $Tr(P) = Tr(Q)$.

Example 19.3 (One-place buffer)

$B(in, out) = in.\overline{out}.B(in, out)$

$\Rightarrow Tr(B(in, out)) = (in \cdot \overline{out})^* \cdot (in + \varepsilon)$

Trace Equivalence II

Remarks:

- The trace language of $P \in Prc$ is accepted by the LTS of P, interpreted as a (finite or infinite) automaton with initial state P and where every state is final.

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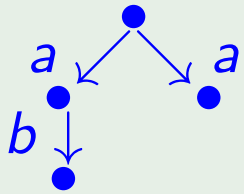
- Trace equivalence is obviously an equivalence relation (i.e., reflexive, symmetric, and transitive).
- Trace equivalence identifies processes with identical LTSs: the trace language of a process consists of the (finite) paths in the LTS. Thus:

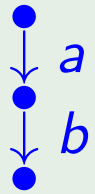
$$LTS(P) = LTS(Q) \Rightarrow Tr(P) = Tr(Q)$$

Trace Equivalence III

Are we satisfied with trace equivalence? No!

Example 19.4

- P : 

and Q : 

  are trace equivalent (since $Tr(P) = Tr(Q) = \{\varepsilon, a, ab\}$)

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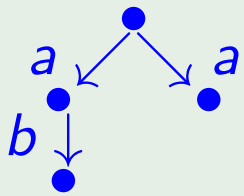
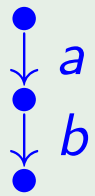
- P : 

and Q : 

are trace equivalent (since $Tr(P) = Tr(Q) = \{\varepsilon, a, ab\}$)
- But P and Q are distinguishable:
  - both can execute ab
  - but P can deny b after a
  - while Q always has to offer b after a

  (e.g., consider a model of vending machine with $a =$ "insert coin", $b =$ "return coffee")

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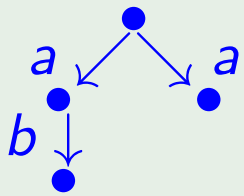
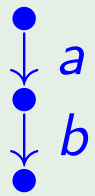
- P : 

and Q : 

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$\Rightarrow$ take into account such deadlock properties

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Definition of Strong Bisimulation I

Observation: equivalence should be sensitive to deadlocks

$\Rightarrow$ needs to take branching structure of processes into account

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$P, Q \in Prc$ are equivalent iff, for every $\alpha \in Act$, every α-successor of P is equivalent to some α-successor of Q, and vice versa.

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- Strong version ignores special function of silent action τ
  (alternative: weak bisimulation; considered later)
- Unidirectional version: simulation
  (not considered here)

Definition of Strong Bisimulation II

Definition 19.5 (Strong bisimulation)

A relation $\rho \subseteq Prc \times Prc$ is called a strong bisimulation if $P \rho Q$ implies, for every $\alpha \in Act$,

1. $P \xrightarrow{\alpha} P' \Rightarrow$ ex. $Q' \in Prc$ such that $Q \xrightarrow{\alpha} Q'$ and $P' \rho Q'$
2. $Q \xrightarrow{\alpha} Q' \Rightarrow$ ex. $P' \in Prc$ such that $P \xrightarrow{\alpha} P'$ and $P' \rho Q'$

$P, Q \in Prc$ are called strongly bisimilar (notation: $P \sim Q$) if there exists a strong bisimulation ρ such that $P \rho Q$.

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Theorem 19.6

$\sim$ is an equivalence relation.

Proof.

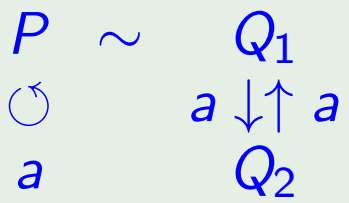
omitted $\square$

Examples

Example 19.7

(on the board)

1. Bisimilar but not LTS equivalent (cf. Example 19.1):



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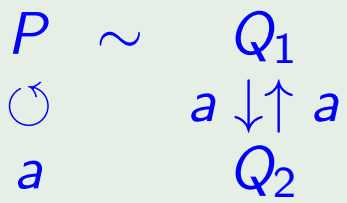
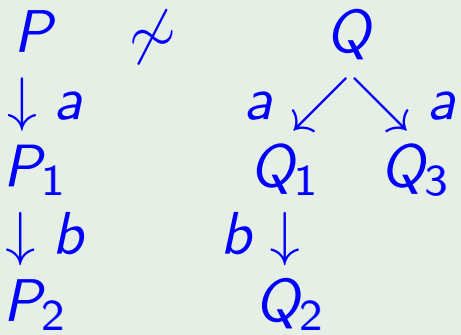


2. Trace equivalent (cf. Example 19.4) but not bisimilar:



Bisimulation vs. LTS/Trace Equivalence

Theorem 19.8

For every $P, Q \in Prc$,

$$LTS(P) = LTS(Q) \quad \begin{array}{c} \Rightarrow \\ \not\Leftarrow \end{array} \quad P \sim Q \quad \begin{array}{c} \Rightarrow \\ \not\Leftarrow \end{array} \quad Tr(P) = Tr(Q)$$

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  (show: $\exists w \in Tr(P) \setminus Tr(Q) \Rightarrow P \not\sim Q$ by induction on $|w|$)

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- $Tr(P) = Tr(Q) \not\Rightarrow P \sim Q$: see Example 19.7(2)

□

More Examples I

Example 19.9

Binary semaphore

(controls exclusive access to two instances of a resource)

Sequential definition:

$$\begin{aligned} Sem_0(get, put) &= get.Sem_1(get, put) \\ Sem_1(get, put) &= get.Sem_2(get, put) + put.Sem_0(get, put) \\ Sem_2(get, put) &= put.Sem_1(get, put) \end{aligned}$$

Parallel definition:

$$\begin{aligned} S(get, put) &= S_0(get, put) \parallel S_0(get, put) \\ S_0(get, put) &= get.S_1(get, put) \\ S_1(get, put) &= put.S_0(get, put) \end{aligned}$$

Proposition: $Sem_0(get, put) \sim S(get, put)$

More Examples II

Example 19.10

Two-place buffer

Sequential definition:

$$\begin{aligned} B_0(in, out) &= in.B_1(in, out) \\ B_1(in, out) &= \overline{out}.B_0(in, out) + in.B_2(in, out) \\ B_2(in, out) &= \overline{out}.B_1(in, out) \end{aligned}$$

Parallel definition:

$$\begin{aligned} B_{\|}(in, out) &= \text{new } com\,(B(in, com) \parallel B(com, out)) \\ B(in, out) &= in.\overline{out}.B(in, out) \end{aligned}$$

Proposition: $B_0(in, out) \not\sim B_{\|}(in, out)$