Semantics and Verification of Software

Lecture 20: Nondeterminism and Parallelism V (Wrap-Up)

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(Software Modeling and Verification)

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Summer Semester 2013

Online Registration for Seminars and Practical Courses (Praktika) in Winter Term 2013/14

Who?

Students of:
- Master Courses
- Bachelor Informatik (~~Pro~~Seminar!)

Where?

www.graphics.rwth-aachen.de/apse

When?

05.07.2013 - 17.07.2013

Outline

1. Recapitulation: Calculus of Communicating Systems
2. Decidability of Strong Bisimulation
3. Definition of Weak Bisimulation
4. Summary: Nondeterminism and Concurrency
5. Further Topics in Formal Semantics
6. Miscellaneous

Semantics of CCS I

Definition (Semantics of CCS)

A process definition $(A_i(a_{i1}, \ldots, a_{in_i}) = P_i \mid 1 \leq i \leq k)$ determines the labeled transition system (LTS) $(Prc, Act, \longrightarrow)$ whose transitions can be inferred from the following rules ($P, P', Q, Q' \in Prc$, $\alpha \in Act$, $\lambda \in N \cup \overline{N}$, $a, b \in N$, $A \in Pid$):

$$(\text{Act})\ \frac{}{\alpha.P \xrightarrow{\alpha} P} \qquad (\text{Com})\ \frac{P \xrightarrow{\lambda} P' \quad Q \xrightarrow{\overline{\lambda}} Q'}{P \parallel Q \xrightarrow{\tau} P' \parallel Q'}$$

$$(\text{Sum}_1)\ \frac{P \xrightarrow{\alpha} P'}{P + Q \xrightarrow{\alpha} P'} \qquad (\text{Sum}_2)\ \frac{Q \xrightarrow{\alpha} Q'}{P + Q \xrightarrow{\alpha} Q'}$$

$$(\text{Par}_1)\ \frac{P \xrightarrow{\alpha} P'}{P \parallel Q \xrightarrow{\alpha} P' \parallel Q} \qquad (\text{Par}_2)\ \frac{Q \xrightarrow{\alpha} Q'}{P \parallel Q \xrightarrow{\alpha} P \parallel Q'}$$

$$(\text{New})\ \frac{P \xrightarrow{\alpha} P' \ (\alpha \notin \{a, \overline{a}\})}{\text{new } a\, P \xrightarrow{\alpha} \text{new } a\, P'} \qquad (\text{Call})\ \frac{P[\vec{a} \mapsto \vec{b}] \xrightarrow{\alpha} P'}{A(\vec{b}) \xrightarrow{\alpha} P'} \text{ if } A(\vec{a}) = P$$

(Here $P[\vec{a} \mapsto \vec{b}]$ denotes the replacement of every a_i by b_i in P.)

Definition of Strong Bisimulation

Definition (Strong bisimulation)

A relation $\rho \subseteq Prc \times Prc$ is called a strong bisimulation if $P \rho Q$ implies, for every $\alpha \in Act$,

1. $P \xrightarrow{\alpha} P' \Rightarrow$ ex. $Q' \in Prc$ such that $Q \xrightarrow{\alpha} Q'$ and $P' \rho Q'$
2. $Q \xrightarrow{\alpha} Q' \Rightarrow$ ex. $P' \in Prc$ such that $P \xrightarrow{\alpha} P'$ and $P' \rho Q'$

$P, Q \in Prc$ are called strongly bisimilar (notation: $P \sim Q$) if there exists a strong bisimulation ρ such that $P \rho Q$.

Theorem

$\sim$ is an equivalence relation.

Proof.

omitted $\square$

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The Problem

We now show that the word problem for strong bisimulation

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Given: $P, Q \in Prc$

Question: $P \sim Q$?

is decidable for finite-state processes (i.e., for those with $|Prc(P)|, |Prc(Q)| < \infty$ where $Prc(P) := \{P' \in Prc \mid P \longrightarrow P'\}$)

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To this aim we give an algorithm which iteratively partitions the state set of an LTS such that the single blocks correspond to the $\sim$-equivalence classes.

The Partitioning Algorithm I

Theorem 20.1 (Partitioning algorithm for $\sim$)

Input: *LTS* $(S, Act, \longrightarrow)$ *(*S *finite)*

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Then, for every $P, Q \in S$*,*

$$P \sim Q \iff \text{ex. } B \in \hat{\Pi} \text{ with } P, Q \in B$$

The Partitioning Algorithm II

Remark: if states from two disjoint LTSs $(S_1, Act_1, \longrightarrow_1)$ and $(S_2, Act_2, \longrightarrow_2)$ (where $S_1 \cap S_2 = \emptyset$) are to be compared, their union $(S_1 \cup S_2, Act_1 \cup Act_2, \longrightarrow_1 \cup \longrightarrow_2)$ is chosen as input (here usually $Act_1 = Act_2$)

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Example 20.2

Binary semaphore (on the board)

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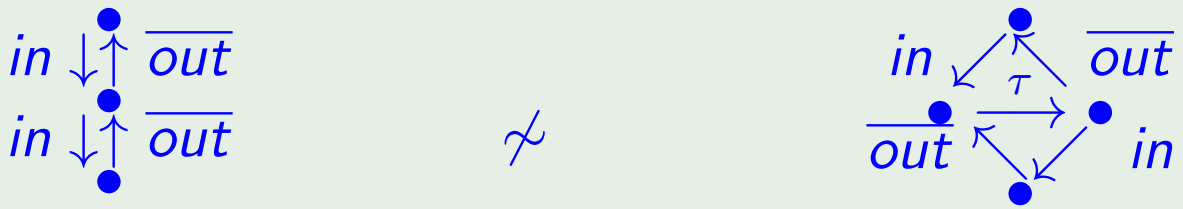
Inadequacy of Strong Bisimulation

Observation: requirement of exact matching sometimes too strong

Example 20.3

Sequential and parallel two-place buffer:

$$
\begin{aligned}
B_0(in, out) &= in.B_1(in, out) \\
B_1(in, out) &= \overline{out}.B_0(in, out) + \\
&\quad\ in.B_2(in, out) \\
B_2(in, out) &= \overline{out}.B_1(in, out)
\end{aligned}
\qquad
\begin{aligned}
B_{\|}(in, out) &= \text{new } com\,(B(in, com) \,\|\, \\
&\qquad\qquad B(com, out)) \\
B(in, out) &= in.\overline{out}.B(in, out)
\end{aligned}
$$



Definition of Weak Bisimulation I

Idea: abstract from silent actions

Definition 20.4

- Given $w \in Act^*$, $\widehat{w} \in (N \cup \overline{N})^*$ denotes the sequence of non-τ-actions in w (in particular, $\widehat{\tau^n} = \varepsilon$ for every $n \in \mathbb{N}$).

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$$P \stackrel{w}{\Longrightarrow} Q \iff P \, (\stackrel{\tau}{\longrightarrow})^* \stackrel{\alpha_1}{\longrightarrow} (\stackrel{\tau}{\longrightarrow})^* \dots (\stackrel{\tau}{\longrightarrow})^* \stackrel{\alpha_n}{\longrightarrow} (\stackrel{\tau}{\longrightarrow})^* \, Q$$

(and hence: $\stackrel{\varepsilon}{\Longrightarrow} = (\stackrel{\tau}{\longrightarrow})^*$).

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  (and hence: $\stackrel{\varepsilon}{\Longrightarrow} = \left(\stackrel{\tau}{\longrightarrow}\right)^*$).
- A relation $\rho \subseteq Prc \times Prc$ is called a weak bisimulation if $P \rho Q$ implies, for every $\alpha \in Act$,
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Definition of Weak Bisimulation II

Remark: each of the two clauses in the definition of weak bisimulation subsumes two cases:

- $P \xrightarrow{\alpha} P'$ where $\alpha \neq \tau$
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Example 20.5

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- Requires new notions of program/process equivalence (bisimulation)

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  - term t over (base and defined) function calls and $x_1, \ldots, x_n$
- Operational semantics (only function calls)
  - call-by-value case:

$$\frac{t_1 \to z_1 \quad \ldots \quad t_n \to z_n \quad t[x_1 \mapsto z_1, \ldots, x_n \mapsto z_n] \to z}{f(t_1, \ldots, t_n) \to z}$$

  - call-by-name case:

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Semantics of Functional Languages II

- Denotational semantics
  - program = equation system (for functions)
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  - monotonic and continuous w.r.t. graph inclusion
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- see [Winskel 1996, Sct. 9] and Functional Programming course [Giesl]

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- Teaching in Winter 2013/14:
  - Course Introduction to Model Checking [Katoen]
  - Course Concurrency Theory [Katoen/Noll]
  - Seminar Trends in Computer-Aided Verification [Katoen/Noll/NN]