

# Verification of Pointer Programs

Stefan Rieger

MOVES: Software Modeling and Verification  
RWTH Aachen University, Germany

23.09.2009



# Pointers

Pointers are indispensable and omnipresent

- object-oriented programming
- dynamic memory management and data structures
- data bases and index structures
- ...

# Pointers

Pointers are indispensable and omnipresent

- object-oriented programming
- dynamic memory management and data structures
- data bases and index structures
- ...

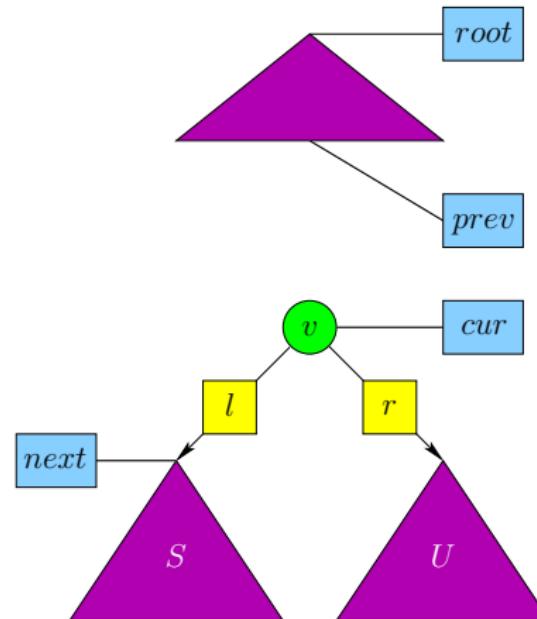
Difficulties

- aliasing creates dependencies
- destructive updates
- dereferencing invalid/null pointers

⇒ automatic verification desirable

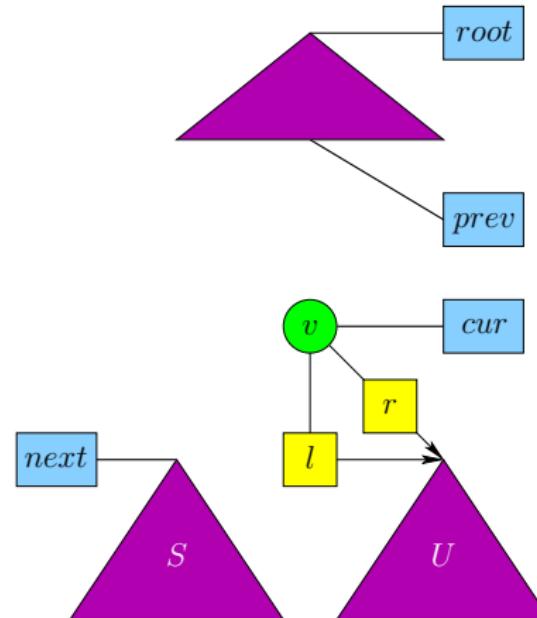
## Example: The Deutsch-Schorr-Waite Algorithm

```
1 if root = null goto 15;  
2 new(sen);  
3 prev := sen;  
4 cur := root;  
5 next := cur.l;  
6 cur.l := cur.r;    ←  
7 cur.r := prev;  
8 prev := cur;  
9 cur := next;  
10 if (cur = sen) goto 15;  
11 if (cur ≠ null) goto 5;  
12 cur := prev;  
13 prev := null;  
14 goto 5;
```



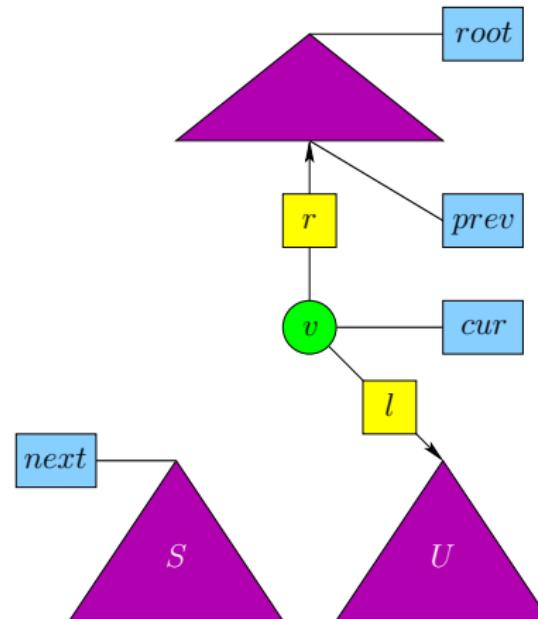
## Example: The Deutsch-Schorr-Waite Algorithm

```
1 if root = null goto 15;  
2 new(sen);  
3 prev := sen;  
4 cur := root;  
5 next := cur.l;  
6 cur.l := cur.r;  
7 cur.r := prev; ←  
8 prev := cur;  
9 cur := next;  
10 if (cur = sen) goto 15;  
11 if (cur ≠ null) goto 5;  
12 cur := prev;  
13 prev := null;  
14 goto 5;
```



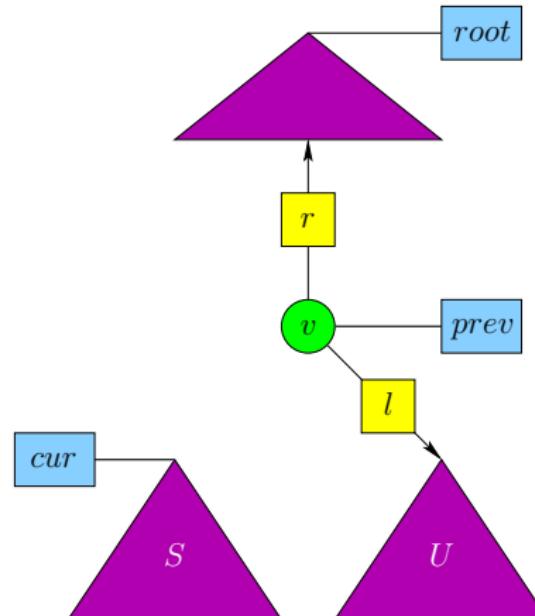
## Example: The Deutsch-Schorr-Waite Algorithm

```
1 if root = null goto 15;  
2 new(sen);  
3 prev := sen;  
4 cur := root;  
5 next := cur.l;  
6 cur.l := cur.r;  
7 cur.r := prev;  
8 prev := cur;      ←  
9 cur := next;  
10 if (cur = sen) goto 15;  
11 if (cur ≠ null) goto 5;  
12 cur := prev;  
13 prev := null;  
14 goto 5;
```



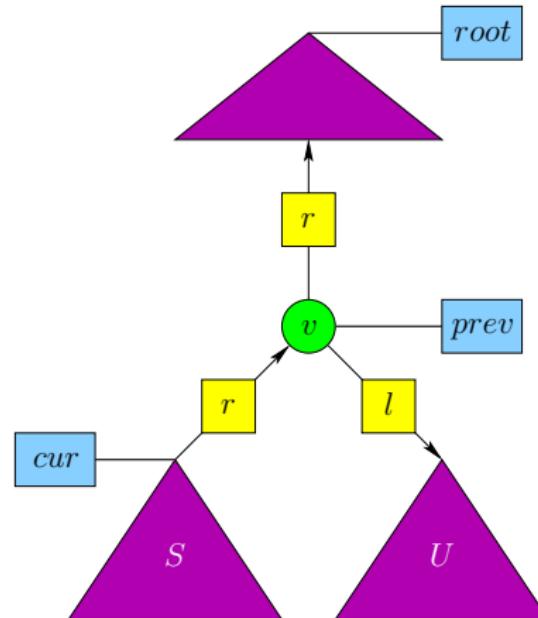
## Example: The Deutsch-Schorr-Waite Algorithm

```
1 if root = null goto 15;  
2 new(sen);  
3 prev := sen;  
4 cur := root;  
5 next := cur.l;  
6 cur.l := cur.r;    ←  
7 cur.r := prev;  
8 prev := cur;  
9 cur := next;  
10 if (cur = sen) goto 15;  
11 if (cur ≠ null) goto 5;  
12 cur := prev;  
13 prev := null;  
14 goto 5;
```



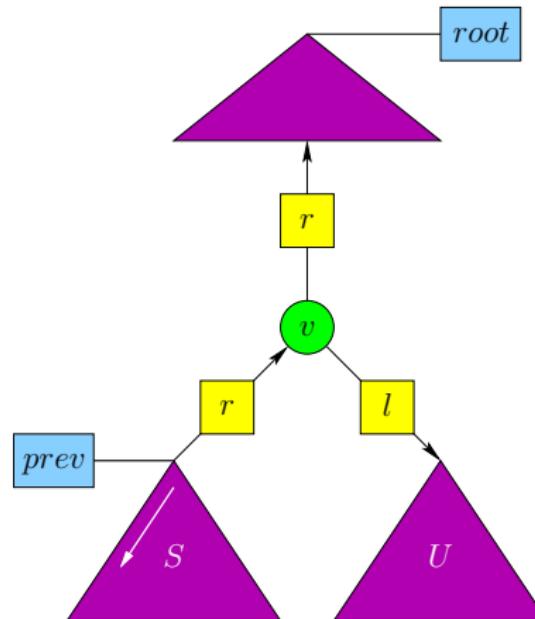
## Example: The Deutsch-Schorr-Waite Algorithm

```
1 if root = null goto 15;  
2 new(sen);  
3 prev := sen;  
4 cur := root;  
5 next := cur.l;  
6 cur.l := cur.r;  
7 cur.r := prev;  
8 prev := cur;      ←  
9 cur := next;  
10 if (cur = sen) goto 15;  
11 if (cur ≠ null) goto 5;  
12 cur := prev;  
13 prev := null;  
14 goto 5;
```



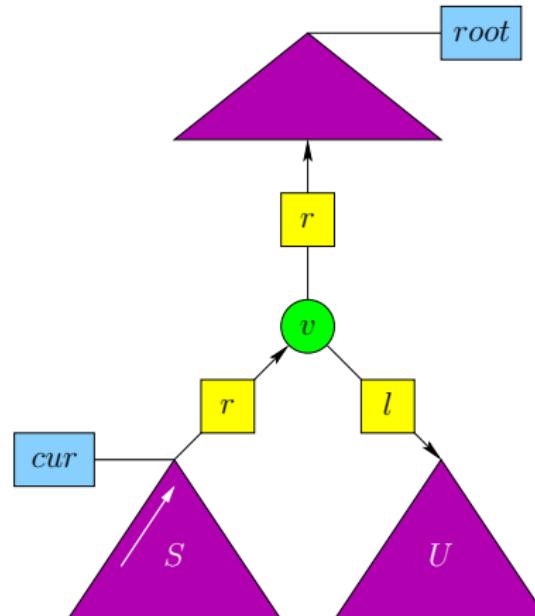
## Example: The Deutsch-Schorr-Waite Algorithm

```
1 if root = null goto 15;  
2 new(sen);  
3 prev := sen;  
4 cur := root;  
5 next := cur.l;  
6 cur.l := cur.r;    ←  
7 cur.r := prev;  
8 prev := cur;  
9 cur := next;  
10 if (cur = sen) goto 15;  
11 if (cur ≠ null) goto 5;  
12 cur := prev;  
13 prev := null;  
14 goto 5;
```



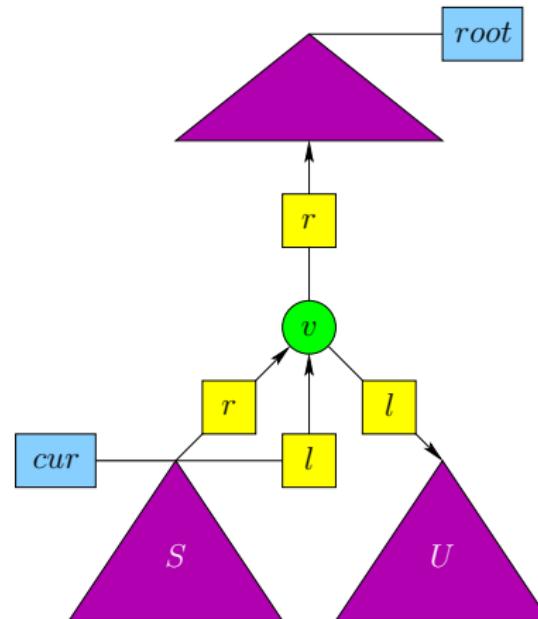
## Example: The Deutsch-Schorr-Waite Algorithm

```
1 if root = null goto 15;  
2 new(sen);  
3 prev := sen;  
4 cur := root;  
5 next := cur.l;  
6 cur.l := cur.r;    ←  
7 cur.r := prev;  
8 prev := cur;  
9 cur := next;  
10 if (cur = sen) goto 15;  
11 if (cur ≠ null) goto 5;  
12 cur := prev;  
13 prev := null;  
14 goto 5;
```



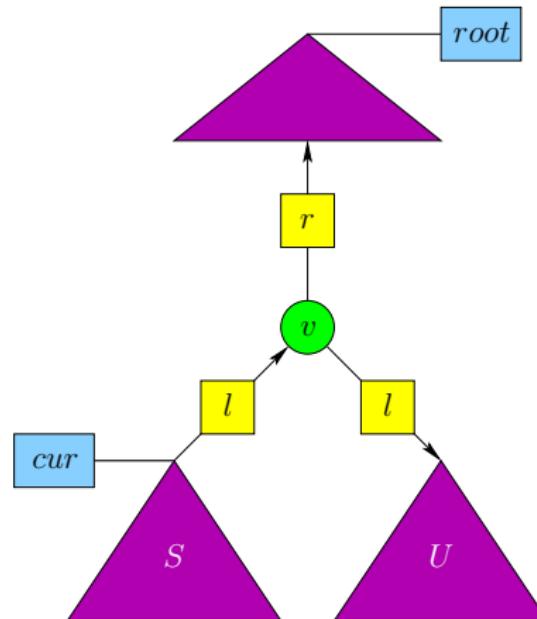
## Example: The Deutsch-Schorr-Waite Algorithm

```
1 if root = null goto 15;  
2 new(sen);  
3 prev := sen;  
4 cur := root;  
5 next := cur.l;  
6 cur.l := cur.r;  
7 cur.r := prev; ←  
8 prev := cur;  
9 cur := next;  
10 if (cur = sen) goto 15;  
11 if (cur ≠ null) goto 5;  
12 cur := prev;  
13 prev := null;  
14 goto 5;
```



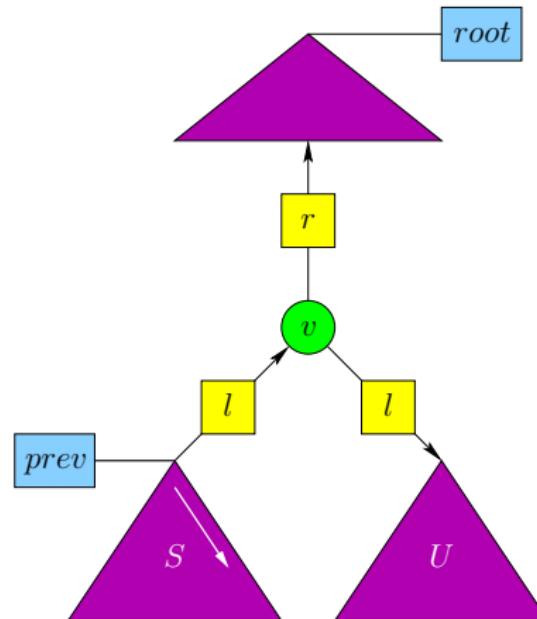
## Example: The Deutsch-Schorr-Waite Algorithm

```
1 if root = null goto 15;  
2 new(sen);  
3 prev := sen;  
4 cur := root;  
5 next := cur.l;  
6 cur.l := cur.r;  
7 cur.r := prev;  
8 prev := cur;      ←  
9 cur := next;  
10 if (cur = sen) goto 15;  
11 if (cur ≠ null) goto 5;  
12 cur := prev;  
13 prev := null;  
14 goto 5;
```



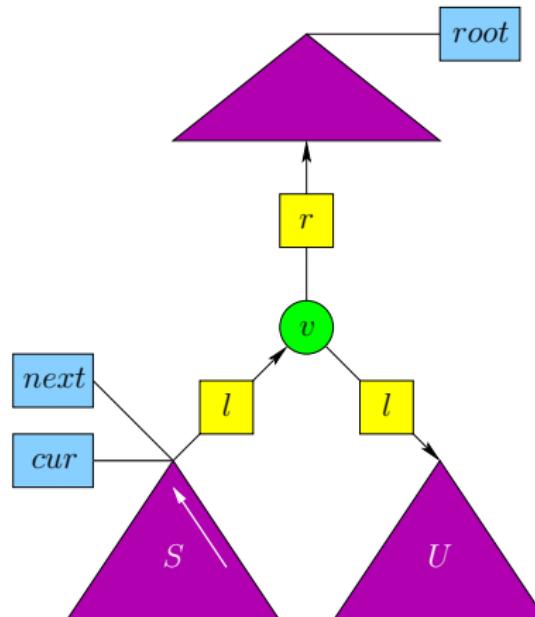
## Example: The Deutsch-Schorr-Waite Algorithm

```
1 if root = null goto 15;  
2 new(sen);  
3 prev := sen;  
4 cur := root;  
5 next := cur.l;  
6 cur.l := cur.r;    ←  
7 cur.r := prev;  
8 prev := cur;  
9 cur := next;  
10 if (cur = sen) goto 15;  
11 if (cur ≠ null) goto 5;  
12 cur := prev;  
13 prev := null;  
14 goto 5;
```



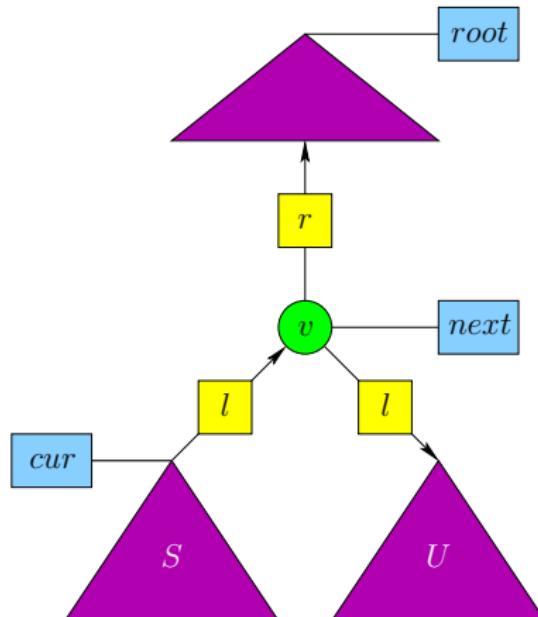
## Example: The Deutsch-Schorr-Waite Algorithm

```
1 if root = null goto 15;  
2 new(sen);  
3 prev := sen;  
4 cur := root;  
5 next := cur.l;    ←  
6 cur.l := cur.r;  
7 cur.r := prev;  
8 prev := cur;  
9 cur := next;  
10 if (cur = sen) goto 15;  
11 if (cur ≠ null) goto 5;  
12 cur := prev;  
13 prev := null;  
14 goto 5;
```



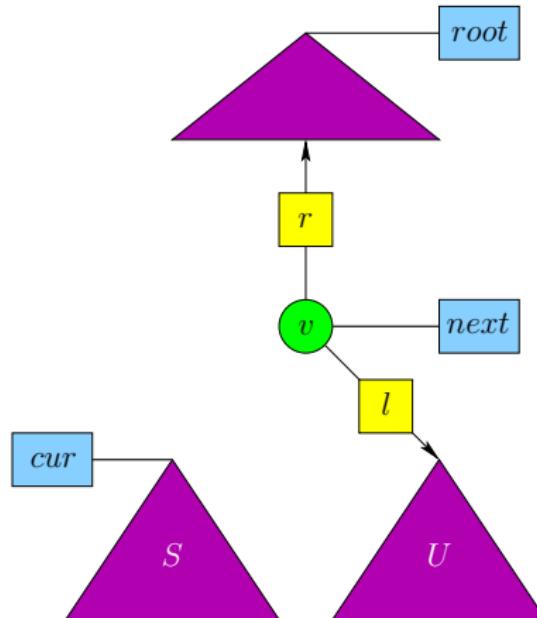
## Example: The Deutsch-Schorr-Waite Algorithm

```
1 if root = null goto 15;  
2 new(sen);  
3 prev := sen;  
4 cur := root;  
5 next := cur.l;  
6 cur.l := cur.r;    ←  
7 cur.r := prev;  
8 prev := cur;  
9 cur := next;  
10 if (cur = sen) goto 15;  
11 if (cur ≠ null) goto 5;  
12 cur := prev;  
13 prev := null;  
14 goto 5;
```



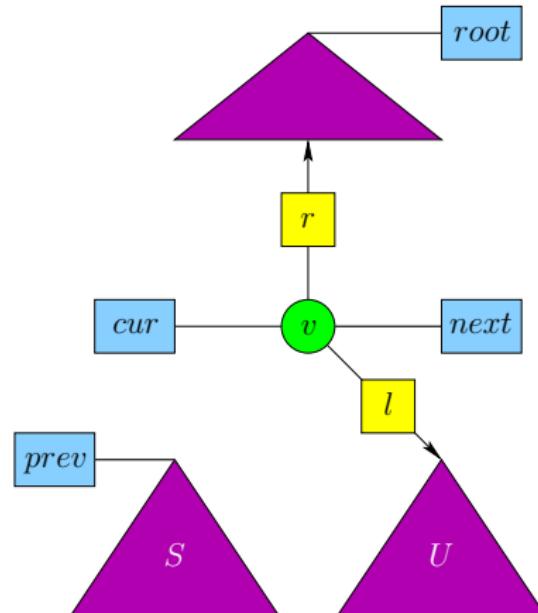
## Example: The Deutsch-Schorr-Waite Algorithm

```
1 if root = null goto 15;  
2 new(sen);  
3 prev := sen;  
4 cur := root;  
5 next := cur.l;  
6 cur.l := cur.r;  
7 cur.r := prev;  
8 prev := cur;      ←  
9 cur := next;  
10 if (cur = sen) goto 15;  
11 if (cur ≠ null) goto 5;  
12 cur := prev;  
13 prev := null;  
14 goto 5;
```



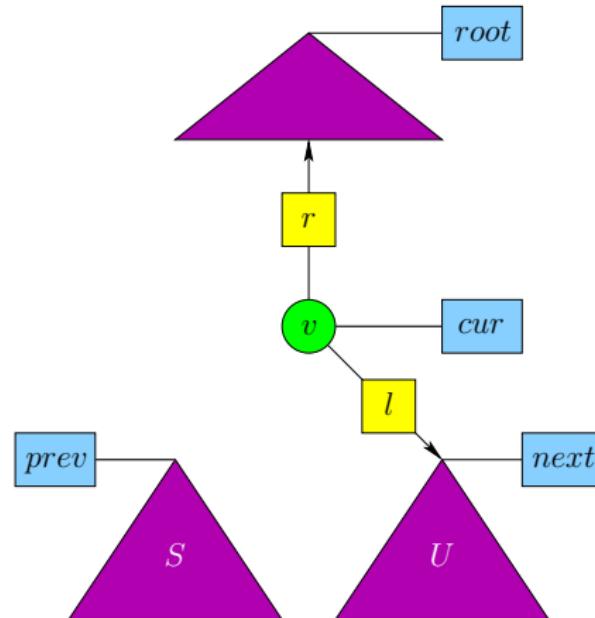
## Example: The Deutsch-Schorr-Waite Algorithm

```
1 if root = null goto 15;  
2 new(sen);  
3 prev := sen;  
4 cur := root;  
5 next := cur.l;    ←  
6 cur.l := cur.r;  
7 cur.r := prev;  
8 prev := cur;  
9 cur := next;  
10 if (cur = sen) goto 15;  
11 if (cur ≠ null) goto 5;  
12 cur := prev;  
13 prev := null;  
14 goto 5;
```



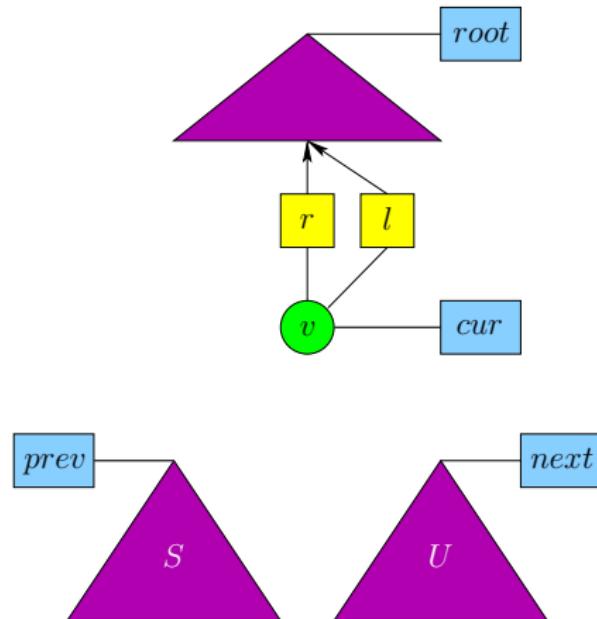
## Example: The Deutsch-Schorr-Waite Algorithm

```
1 if root = null goto 15;  
2 new(sen);  
3 prev := sen;  
4 cur := root;  
5 next := cur.l;  
6 cur.l := cur.r;    ←  
7 cur.r := prev;  
8 prev := cur;  
9 cur := next;  
10 if (cur = sen) goto 15;  
11 if (cur ≠ null) goto 5;  
12 cur := prev;  
13 prev := null;  
14 goto 5;
```



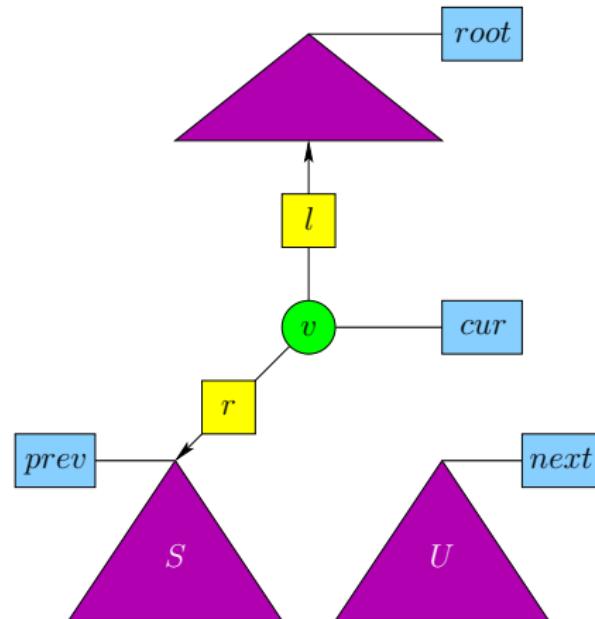
## Example: The Deutsch-Schorr-Waite Algorithm

```
1 if root = null goto 15;  
2 new(sen);  
3 prev := sen;  
4 cur := root;  
5 next := cur.l;  
6 cur.l := cur.r;  
7 cur.r := prev; ←  
8 prev := cur;  
9 cur := next;  
10 if (cur = sen) goto 15;  
11 if (cur ≠ null) goto 5;  
12 cur := prev;  
13 prev := null;  
14 goto 5;
```



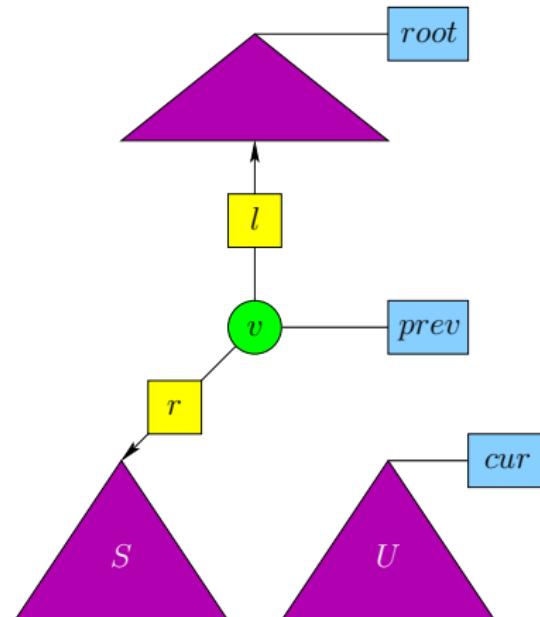
## Example: The Deutsch-Schorr-Waite Algorithm

```
1 if root = null goto 15;  
2 new(sen);  
3 prev := sen;  
4 cur := root;  
5 next := cur.l;  
6 cur.l := cur.r;  
7 cur.r := prev;  
8 prev := cur;      ←  
9 cur := next;  
10 if (cur = sen) goto 15;  
11 if (cur ≠ null) goto 5;  
12 cur := prev;  
13 prev := null;  
14 goto 5;
```



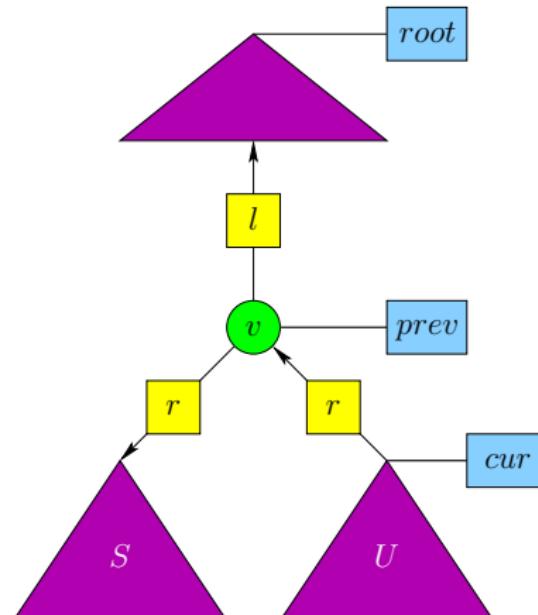
## Example: The Deutsch-Schorr-Waite Algorithm

```
1 if root = null goto 15;  
2 new(sen);  
3 prev := sen;  
4 cur := root;  
5 next := cur.l;  
6 cur.l := cur.r; ←  
7 cur.r := prev;  
8 prev := cur;  
9 cur := next;  
10 if (cur = sen) goto 15;  
11 if (cur ≠ null) goto 5;  
12 cur := prev;  
13 prev := null;  
14 goto 5;
```



## Example: The Deutsch-Schorr-Waite Algorithm

```
1 if root = null goto 15;  
2 new(sen);  
3 prev := sen;  
4 cur := root;  
5 next := cur.l;  
6 cur.l := cur.r;  
7 cur.r := prev;  
8 prev := cur;      ←  
9 cur := next;  
10 if (cur = sen) goto 15;  
11 if (cur ≠ null) goto 5;  
12 cur := prev;  
13 prev := null;  
14 goto 5;
```

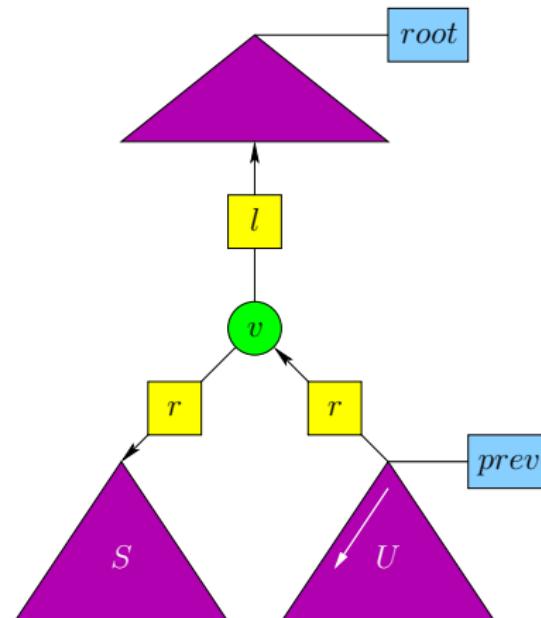


# Example: The Deutsch-Schorr-Waite Algorithm

```

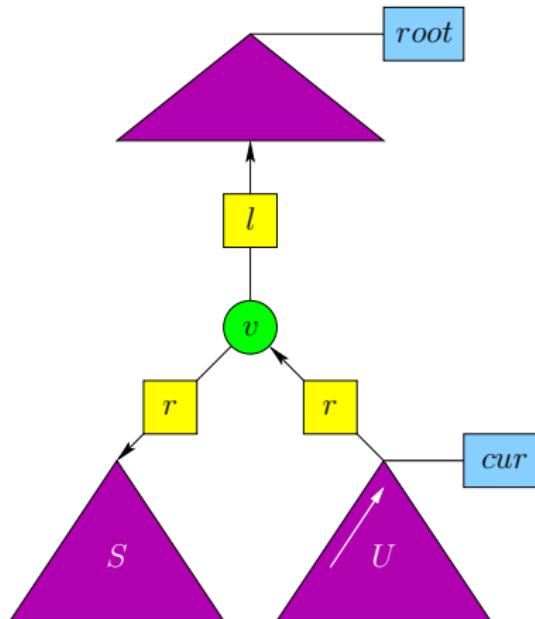
1 if root = null goto 15;
2 new(sen);
3 prev := sen;
4 cur := root;
5 next := cur.l;
6 cur.l := cur.r;    ←
7 cur.r := prev;
8 prev := cur;
9 cur := next;
10 if (cur = sen) goto 15;
11 if (cur ≠ null) goto 5;
12 cur := prev;
13 prev := null;
14 goto 5;

```



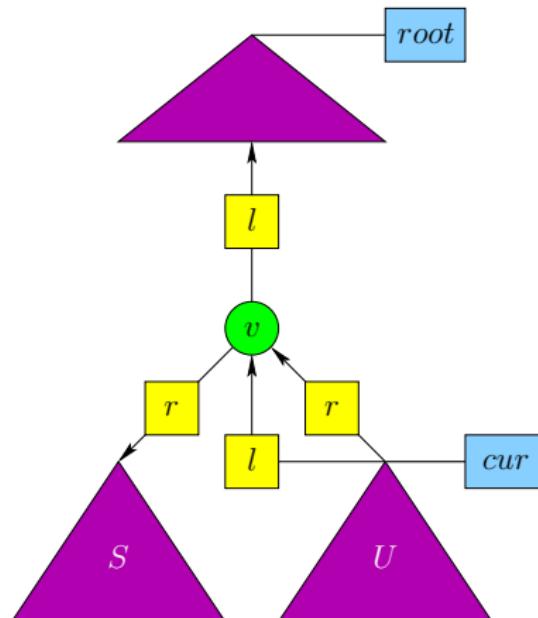
## Example: The Deutsch-Schorr-Waite Algorithm

```
1 if root = null goto 15;  
2 new(sen);  
3 prev := sen;  
4 cur := root;  
5 next := cur.l;  
6 cur.l := cur.r; ←  
7 cur.r := prev;  
8 prev := cur;  
9 cur := next;  
10 if (cur = sen) goto 15;  
11 if (cur ≠ null) goto 5;  
12 cur := prev;  
13 prev := null;  
14 goto 5;
```



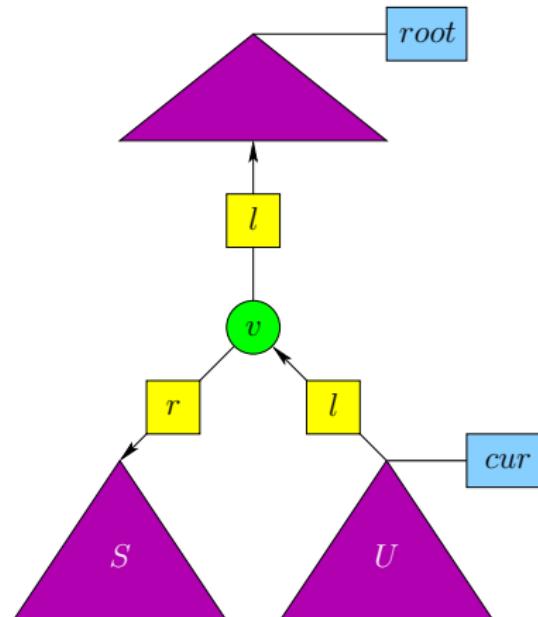
## Example: The Deutsch-Schorr-Waite Algorithm

```
1 if root = null goto 15;  
2 new(sen);  
3 prev := sen;  
4 cur := root;  
5 next := cur.l;  
6 cur.l := cur.r;  
7 cur.r := prev; ←  
8 prev := cur;  
9 cur := next;  
10 if (cur = sen) goto 15;  
11 if (cur ≠ null) goto 5;  
12 cur := prev;  
13 prev := null;  
14 goto 5;
```



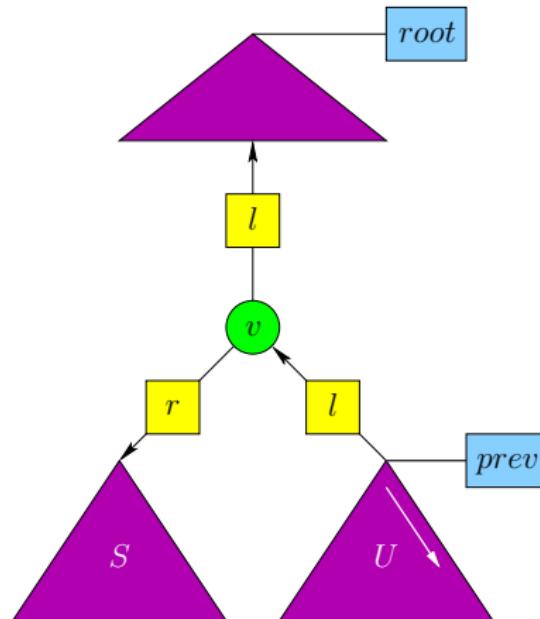
## Example: The Deutsch-Schorr-Waite Algorithm

```
1 if root = null goto 15;  
2 new(sen);  
3 prev := sen;  
4 cur := root;  
5 next := cur.l;  
6 cur.l := cur.r;  
7 cur.r := prev;  
8 prev := cur;      ←  
9 cur := next;  
10 if (cur = sen) goto 15;  
11 if (cur ≠ null) goto 5;  
12 cur := prev;  
13 prev := null;  
14 goto 5;
```



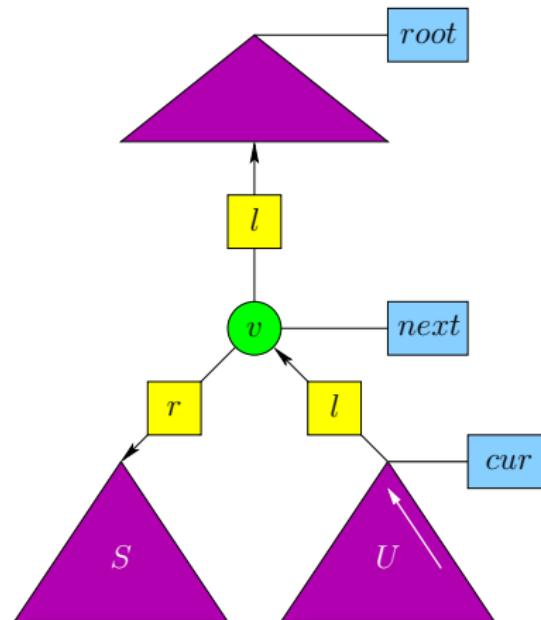
## Example: The Deutsch-Schorr-Waite Algorithm

```
1 if root = null goto 15;  
2 new(sen);  
3 prev := sen;  
4 cur := root;  
5 next := cur.l;  
6 cur.l := cur.r;    ←  
7 cur.r := prev;  
8 prev := cur;  
9 cur := next;  
10 if (cur = sen) goto 15;  
11 if (cur ≠ null) goto 5;  
12 cur := prev;  
13 prev := null;  
14 goto 5;
```



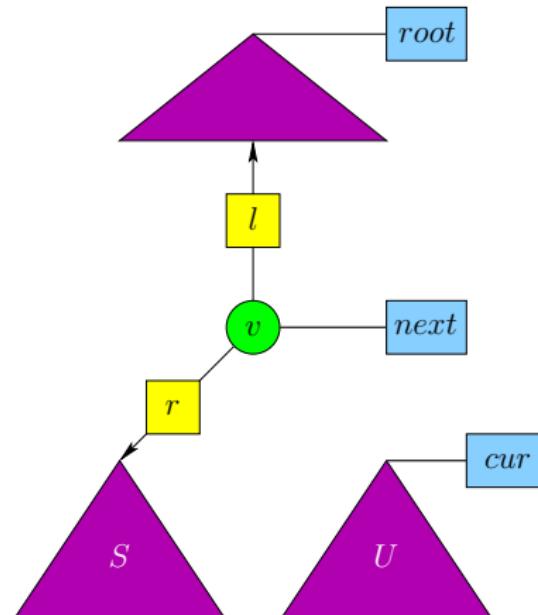
## Example: The Deutsch-Schorr-Waite Algorithm

```
1 if root = null goto 15;  
2 new(sen);  
3 prev := sen;  
4 cur := root;  
5 next := cur.l;  
6 cur.l := cur.r;    ←  
7 cur.r := prev;  
8 prev := cur;  
9 cur := next;  
10 if (cur = sen) goto 15;  
11 if (cur ≠ null) goto 5;  
12 cur := prev;  
13 prev := null;  
14 goto 5;
```



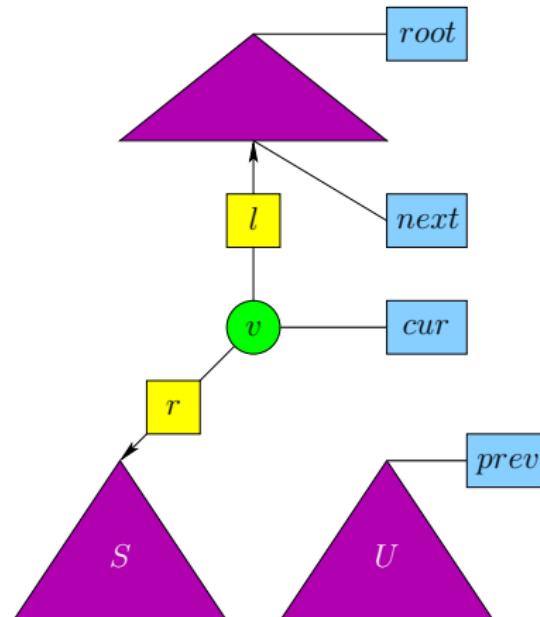
## Example: The Deutsch-Schorr-Waite Algorithm

```
1 if root = null goto 15;  
2 new(sen);  
3 prev := sen;  
4 cur := root;  
5 next := cur.l;  
6 cur.l := cur.r;  
7 cur.r := prev;  
8 prev := cur;      ←  
9 cur := next;  
10 if (cur = sen) goto 15;  
11 if (cur ≠ null) goto 5;  
12 cur := prev;  
13 prev := null;  
14 goto 5;
```



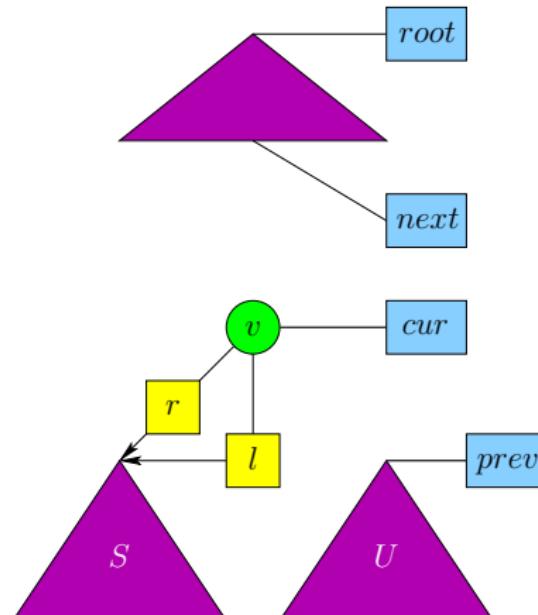
## Example: The Deutsch-Schorr-Waite Algorithm

```
1 if root = null goto 15;  
2 new(sen);  
3 prev := sen;  
4 cur := root;  
5 next := cur.l;  
6 cur.l := cur.r;    ←  
7 cur.r := prev;  
8 prev := cur;  
9 cur := next;  
10 if (cur = sen) goto 15;  
11 if (cur ≠ null) goto 5;  
12 cur := prev;  
13 prev := null;  
14 goto 5;
```



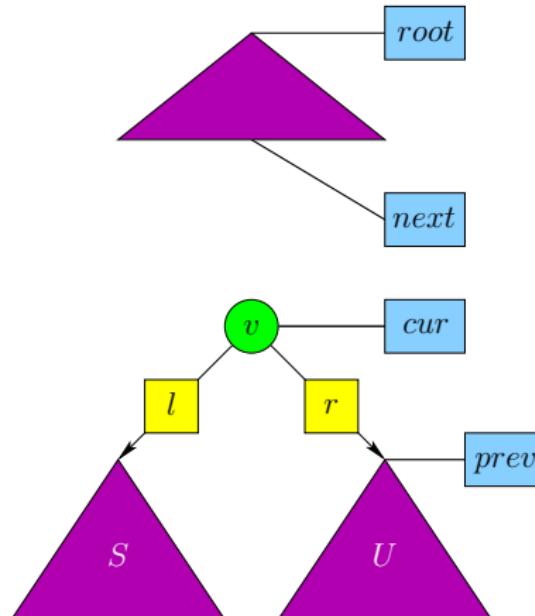
## Example: The Deutsch-Schorr-Waite Algorithm

```
1 if root = null goto 15;  
2 new(sen);  
3 prev := sen;  
4 cur := root;  
5 next := cur.l;  
6 cur.l := cur.r;  
7 cur.r := prev; ←  
8 prev := cur;  
9 cur := next;  
10 if (cur = sen) goto 15;  
11 if (cur ≠ null) goto 5;  
12 cur := prev;  
13 prev := null;  
14 goto 5;
```



## Example: The Deutsch-Schorr-Waite Algorithm

```
1 if root = null goto 15;  
2 new(sen);  
3 prev := sen;  
4 cur := root;  
5 next := cur.l;  
6 cur.l := cur.r;  
7 cur.r := prev;  
8 prev := cur;      ←  
9 cur := next;  
10 if (cur = sen) goto 15;  
11 if (cur ≠ null) goto 5;  
12 cur := prev;  
13 prev := null;  
14 goto 5;
```



# Verification of Pointer Structures

## Problems

- handling inputs of arbitrary size
- dynamic memory allocation at runtime

⇒ possibly **infinite state space**

# Verification of Pointer Structures

## Problems

- handling inputs of arbitrary size
- dynamic memory allocation at runtime

⇒ possibly **infinite state space**

## Approach: Over-Approximation by Abstraction

- use **HRGs** to model data structures
- **abstraction** and **concretization** based on HRG rules

⇒ **finite state spaces** for e.g. model checking

# Verification of Pointer Structures

## Problems

- handling inputs of arbitrary size
- dynamic memory allocation at runtime

⇒ possibly **infinite state space**

## Approach: Over-Approximation by Abstraction

- use **HRGs** to model data structures
- **abstraction** and **concretization** based on HRG rules

⇒ **finite state spaces** for e.g. model checking

## Simple Pointer Programming Language (only pointers as data)

- pointer assignment ( $x.a := y.b$ )
- creation of objects ( $\text{new}(x)$ )
- **limited dereferencing depth** (no real restriction)

## Related Work

**Shape Analysis** represents unbounded heap graphs by three-valued logical structures [Sagiv et al., 2002, Beyer et al., 2006]

**Separation Logic** is an extension of Hoare logic [Reynolds, 2002, O'Hearn et al., 2004]

**Graph Transformation** is used in different approaches:

- abstraction and verification of graph transformation systems [Baldan and König, 2002, Baldan et al., 2004, Kastenberg and Rensink, 2006]
- model pointer assignments directly by graph transformations [Rensink, 2004, Rensink and Distefano, 2006]
- graph reduction grammars [Bakewell et al., 2004a, Bakewell et al., 2004b, Dodds and Plump, 2006]

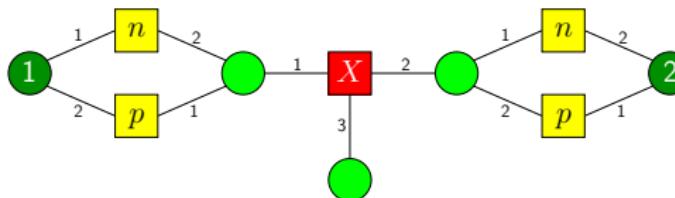
# Alphabets and Hypergraphs

## Ranked Alphabet $\Sigma$

- ranking function  $rk : \Sigma \rightarrow \mathbb{N}$
- $\Sigma$  consists of terminals and nonterminals:  $\Sigma = T_\Sigma \uplus N_\Sigma$

## Hypergraphs

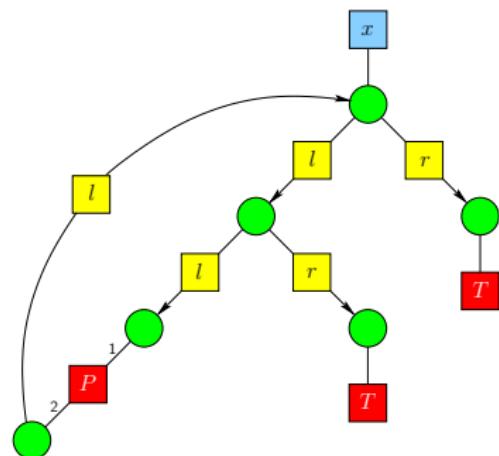
- hyperedges connect an arbitrary number of vertices
- hyperedges are labeled with symbols from  $\Sigma$
- rank of label determines the arity of the edge
- external vertices are used for hyperedge replacements



# Representing Heap States

Heapgraph  $\rightarrow$  Hypergraph

	Rank of Edges	Type of Label
pointers	2	terminal
program variables	1	variable (terminal)
abstract subgraphs	arbitrary	nonterminal



$s$   $\rightarrow$  pointer with selector  $s$

$x$   $\rightarrow$  program variable

$X$   $\rightarrow$  nonterminal edge

omit tentacle numbers when order clear

# Concrete and Abstract Heaps

## Abstract Heap

A heap configuration (=hypergraph) is **abstract**, if it contains at least one nonterminal edge.

# Concrete and Abstract Heaps

## Abstract Heap

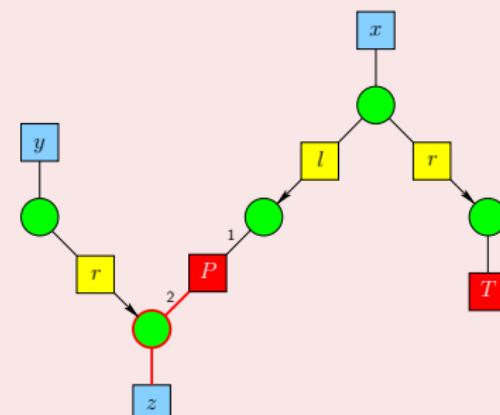
A heap configuration (=hypergraph) is **abstract**, if it contains at least one nonterminal edge.

## Admissibility

A heap configuration is **admissible** if nodes referred by variables are not adjacent to nonterminal edges.

Useful for abstract semantics (“**concrete** assignment”).

## Inadmissible



# Overview

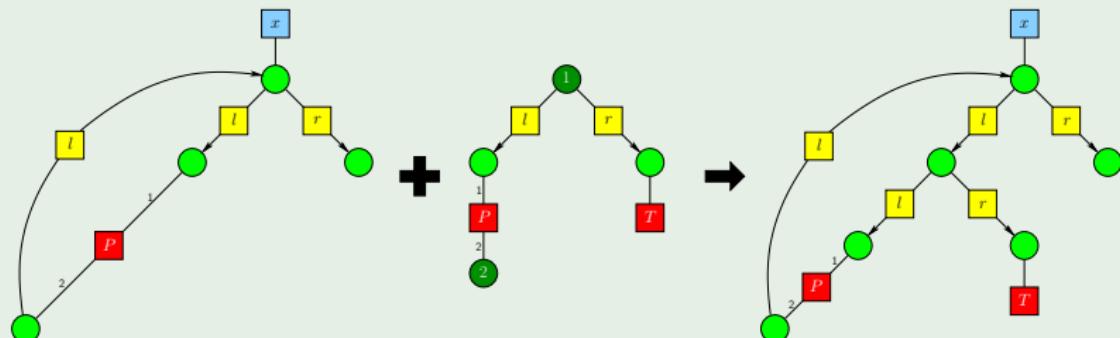
- ① Hyperedge Replacement
- ② Abstraction and Concretization
- ③ Pointer Logic
- ④ Verification and Model Checking

# Hyperedge Replacement

## Executing a hyperedge replacement

- 1 Hypergraph  $H$  with hyperedge  $e \in E_H$  s.t.  $\ell(e) \in N_\Sigma$
- 2 Hypergraph  $R$  with  $|ext_R| = rk(e)$

## Example

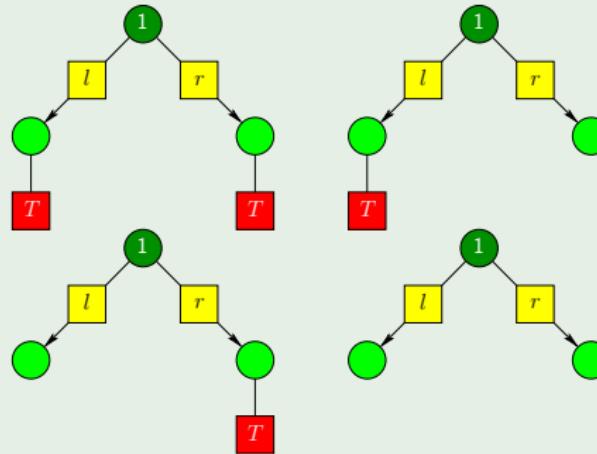


## Hyperedge Replacement Grammars

## Definition

A **HRG**  $G$  is a set of productions of the form  $X \rightarrow R$  with  $X \in N_\Sigma$  and hypergraph  $R$  where  $|ext_R| = rk(X)$ .

## Example: HRG for (fully branched) Binary Trees



## Properties

## Context-freeness

HRGs are **context-free** and **confluent**.

## Applicability

A rule is **applicable** to a hypergraph if it contains a nonterminal that matches the rule's LHS.

## Derivation

A **derivation** is a sequence  $H_0 \xrightarrow{G} H_1 \xrightarrow{G} H_2 \xrightarrow{G} \dots$  where each  $H_i \xrightarrow{G} H_{i+1}$  is an application of a rule from  $G$ .

## Properties

## Context-freeness

HRGs are context-free and confluent.

## Applicability

A rule is **applicable** to a hypergraph if it contains a nonterminal that matches the rule's LHS.

## Derivation

A **derivation** is a sequence  $H_0 \xrightarrow{G} H_1 \xrightarrow{G} H_2 \xrightarrow{G} \dots$  where each  $H_i \xrightarrow{G} H_{i+1}$  is an application of a rule from  $G$ .

## Graph Language of HRG $G$

$$\mathfrak{L}(G, H) = \{K \in \text{HGraph}_{\textcolor{brown}{T}_\Sigma} \mid H \xrightarrow{G} {}^\star K\}$$

(= all **terminal** graphs which are derivable from  $H$ )

# Overview

① Hyperedge Replacement

② Abstraction and Concretization

③ Pointer Logic

④ Verification and Model Checking

# Abstracting the Heap

## Abstraction

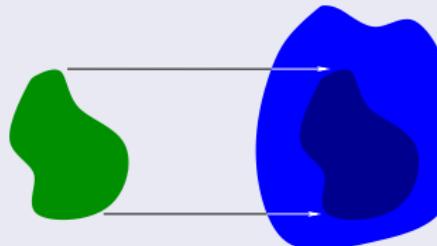
For HRG  $G$  and hypergraph  $H$  the set of **abstractions** of  $H$  is

$$\text{Abstractions}(H) = \{K \in \text{HGraph}_\Sigma \mid K \xrightarrow{G}^+ H\}$$

If  $\text{LHS} < \text{RHS}$  for all rules in  $G$ ,  $\text{Abstractions}(H)$  is finite.

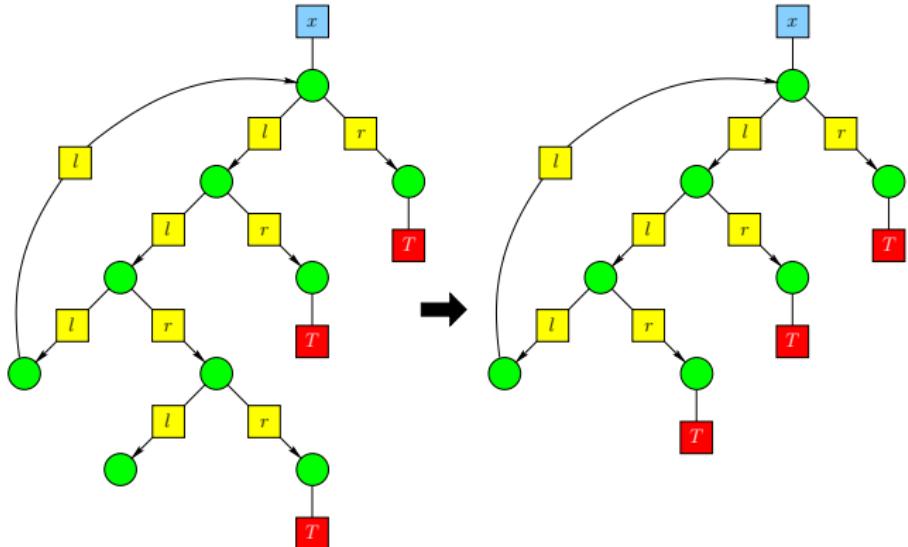
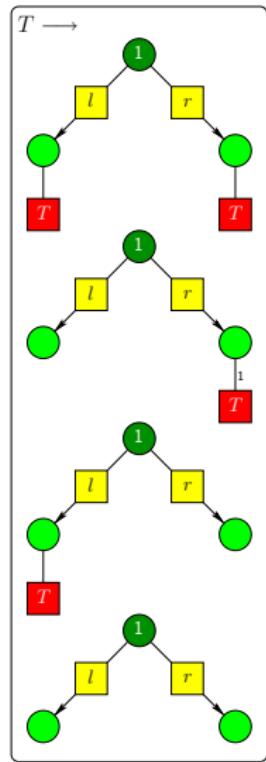
## Idea

- Compute abstractions by **reverse application** of HRG rules
- Reverse application requires finding a **subgraph isomorphism**

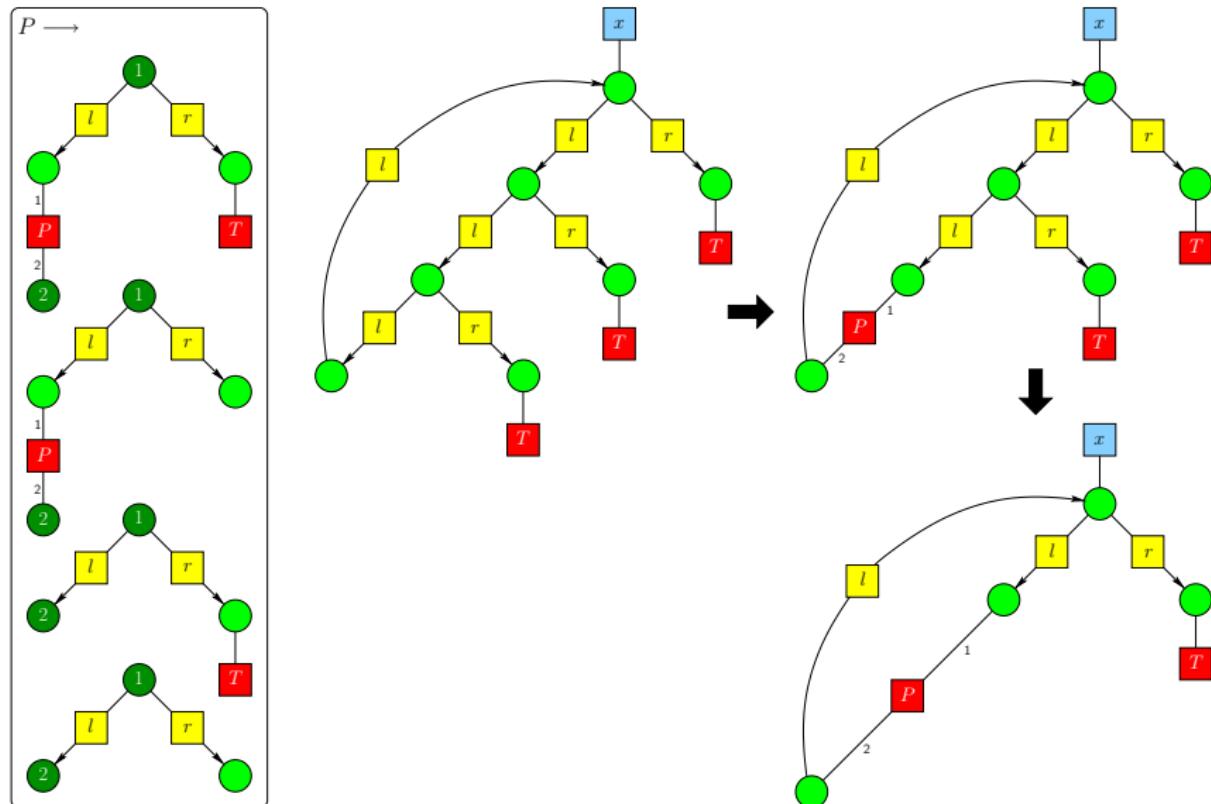


- Reverse rule application is **not confluent**

## Abstraction Example - Binary Trees



## Path Abstraction



# Abstracting the Heap II

## Correctness (but Over-Approximation)

By definition every concrete heap configuration can be regenerated from its abstractions.

$$\text{Abstractions}(H) = \{K \in \text{HGraph}_{\Sigma} \mid K \xrightarrow{g}^+ H\}$$

# Abstracting the Heap II

## Correctness (but Over-Approximation)

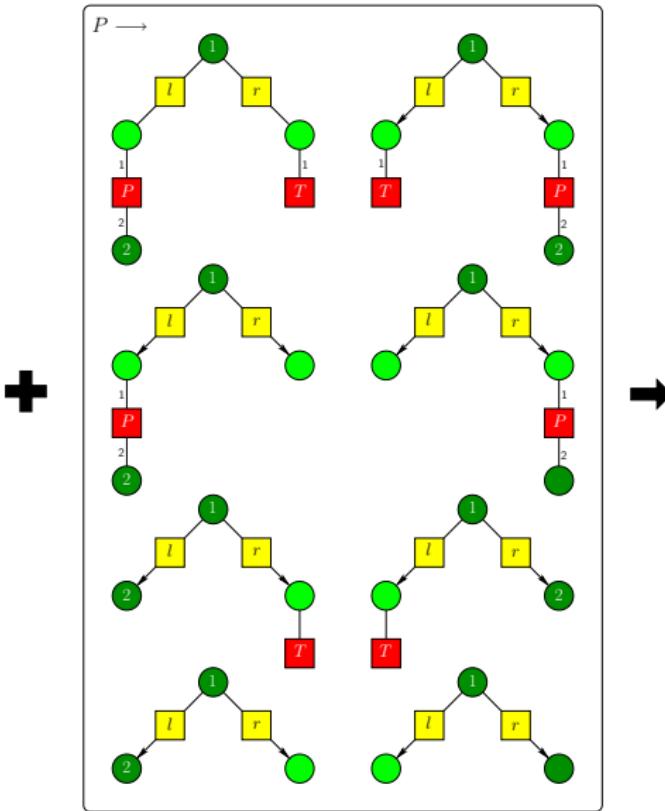
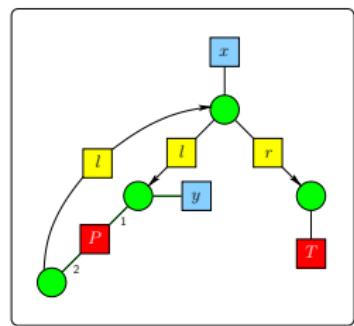
By definition every concrete heap configuration can be regenerated from its abstractions.

$$\text{Abstractions}(H) = \{K \in \text{HGraph}_{\Sigma} \mid K \xrightarrow{g}^+ H\}$$

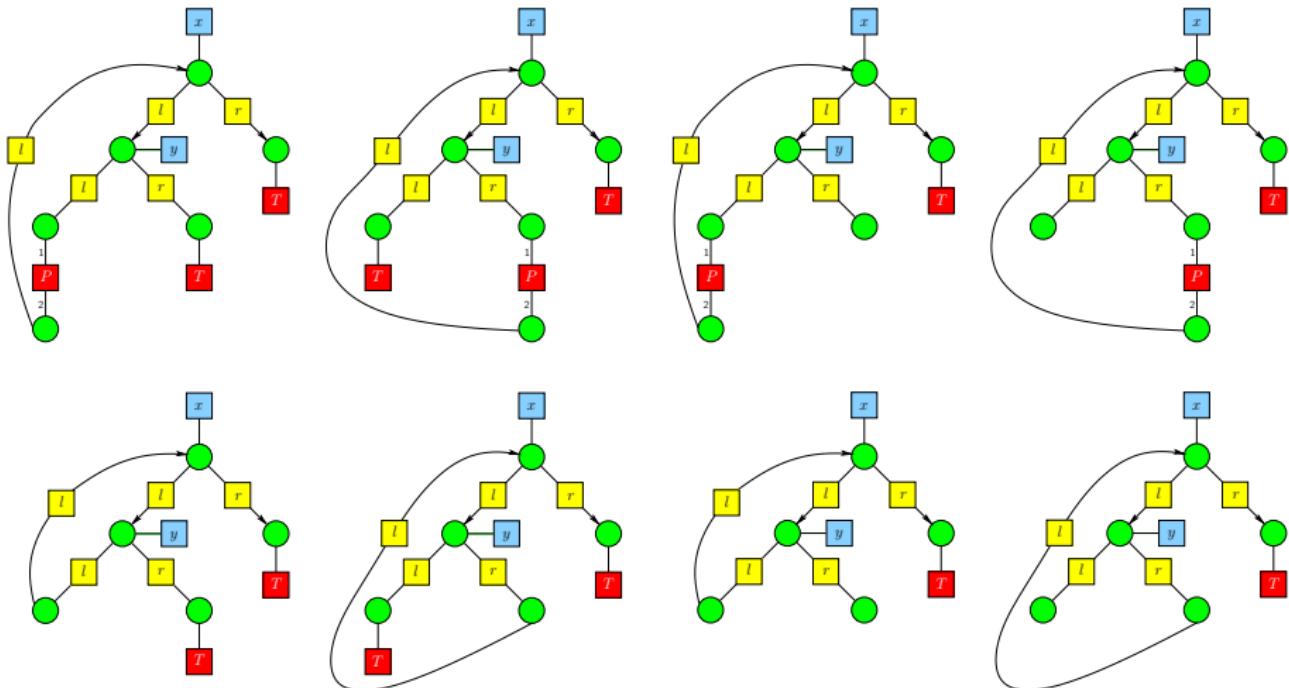
## Abstraction alone insufficient

- Assignment easy since admissibility guarantees concrete edges near variables.
- **But:** assignments may yield inadmissible configurations
- **Idea:** materialize concrete objects from nonterminals (partial concretization)

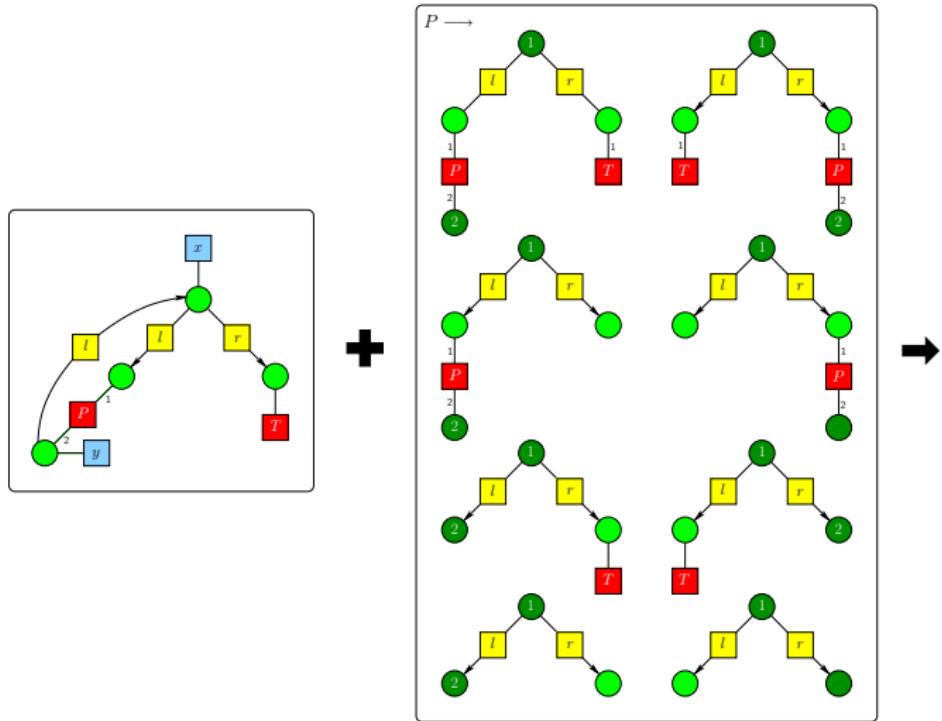
## Partial Concretization by Forward Rule Application



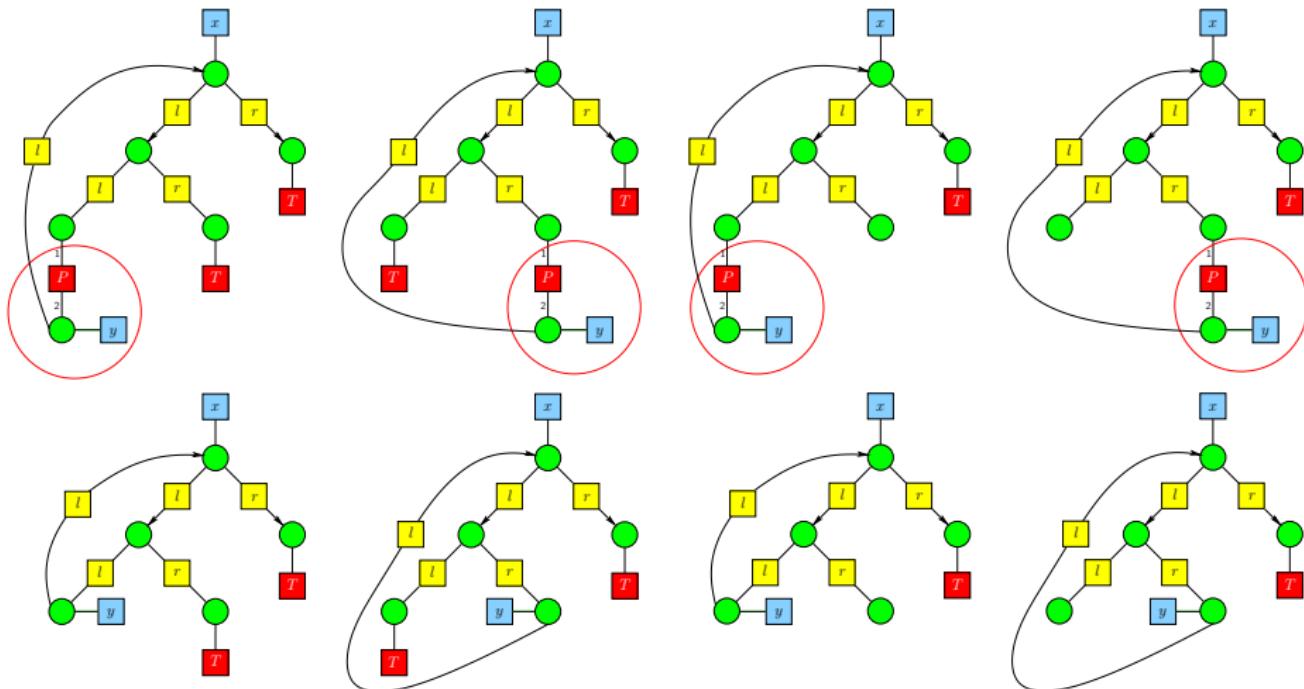
## Resulting Hypergraphs



## Different Situation



## Inadmissible Results

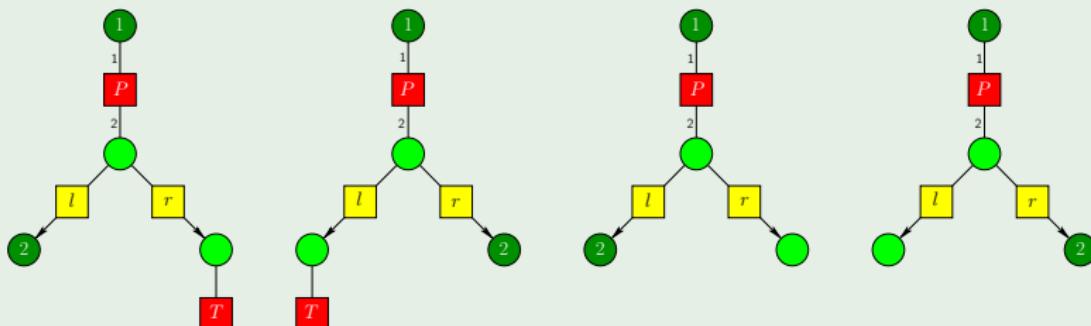


# The Solution

## Heap Abstraction Grammars

- introduce **redundant** rules allowing concretization “from below”
- additional rules must guarantee **completeness**

## Additional Rules



# Overview

- ① Hyperedge Replacement
- ② Abstraction and Concretization
- ③ Pointer Logic
- ④ Verification and Model Checking

## Temporal Pointer Logic

- Combination of LTL operators and pointer comparisons
- Arbitrarily deep dereferencing

## Formal Definition

Let  $\mathfrak{F}$  be a set of flags (err, term, ...).

$$\begin{aligned}
 \text{TPL}(\Sigma, \mathfrak{F}) ::= & \text{ TRUE } | \mathfrak{F} | \text{ DEREF}_\Sigma = \text{ DEREF}_\Sigma \\
 & | \neg \text{TPL}(\Sigma, \mathfrak{F}) | \text{TPL}(\Sigma, \mathfrak{F}) \wedge \text{TPL}(\Sigma, \mathfrak{F}) \\
 & | \mathbf{X} \text{TPL}(\Sigma, \mathfrak{F}) | \text{TPL}(\Sigma, \mathfrak{F}) \mathbf{U} \text{TPL}(\Sigma, \mathfrak{F})
 \end{aligned}$$

$\text{DEREF}_{\Sigma} ::= \text{null} \mid \text{Var}_{\Sigma} \mid \text{DEREF}_{\Sigma} . \text{Sel}_{\Sigma}$

$$\mathbf{F}\varphi \equiv \text{TRUE} \quad \mathbf{U}\varphi \quad \mathbf{G}\varphi \equiv \neg \mathbf{F}\neg\varphi$$

## Semantics of TPL

## Interpretation

- Interpret TPL formulae on infinite **and finite** sequences of **heap configurations**.
- Every trace of heap configurations has an associated trace of (sets of) **flags** of equal length.

## Finite Traces

Let  $t \in \text{aHHC}_{\Sigma}^*$  and  $u \in \mathfrak{F}^*$  be a finite traces of length  $n$ . Implicit extension as follows:

$$\begin{array}{ccccccccc}
 t(1) & t(2) & \dots & t(n) & & t(n) & & t(n) & \dots \\
 u(1) & u(2) & \dots & u(n) & u(n) \cup \{\text{term}\} & u(n) \cup \{\text{term}\} & & & \dots
 \end{array}$$

## Formal Semantics of Pointer Comparisons

## Concrete Semantics

$$\text{CSAT}[\xi = \zeta, H] = \begin{cases} 1 & \mathcal{D}[\xi, H] = \mathcal{D}[\zeta, H] \neq \perp \\ 0 & \text{otherwise} \end{cases}$$

$\mathcal{D}[\zeta, H]$ : “intuitive” concrete expression semantics

# Formal Semantics of Pointer Comparisons

## Concrete Semantics

$$\text{CSAT}[\xi = \zeta, H] = \begin{cases} 1 & \mathcal{D}[\xi, H] = \mathcal{D}[\zeta, H] \neq \perp \\ 0 & \text{otherwise} \end{cases}$$

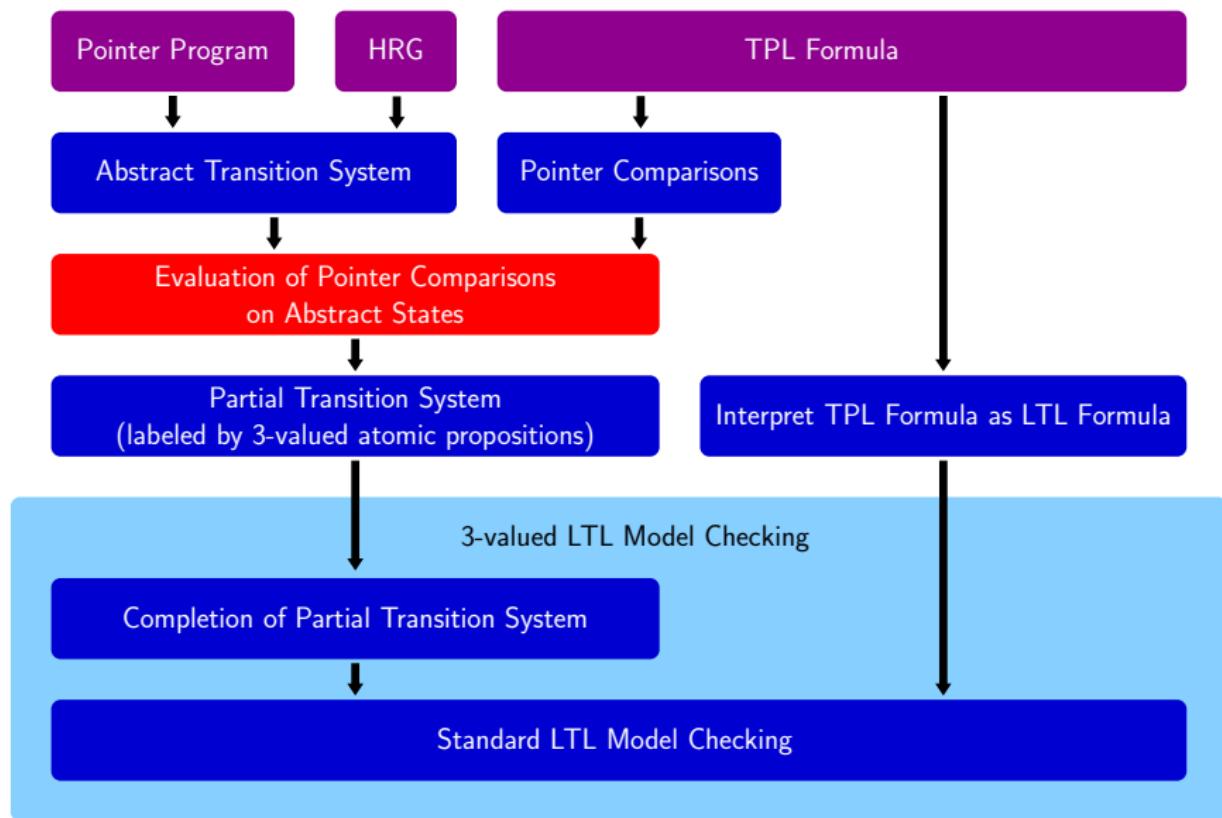
$\mathcal{D}[\zeta, H]$ : “intuitive” concrete expression semantics

## Abstract Semantics – 3 cases

$$\text{ASAT}[\gamma, H] = \begin{cases} 1 & \text{if } \forall H' \in \mathfrak{L}(G, H) : \text{CSAT}[\gamma, H'] = 1 \\ 0 & \text{if } \forall H' \in \mathfrak{L}(G, H) : \text{CSAT}[\gamma, H'] = 0 \\ \frac{1}{2} & \text{otherwise} \end{cases}$$

# Overview

- ① Hyperedge Replacement
- ② Abstraction and Concretization
- ③ Pointer Logic
- ④ Verification and Model Checking

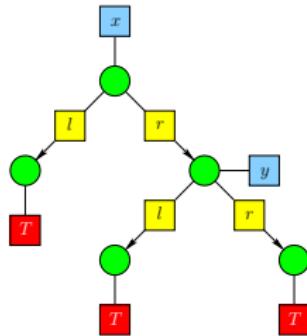


## Evaluating Pointer Comparisons

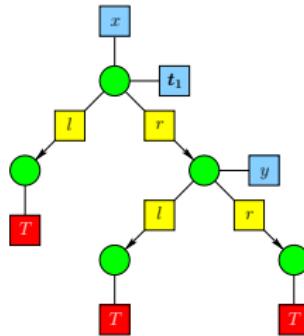
$$x.a_1.a_2...a_m = y.b_1.b_2...b_n$$

- Use two auxiliary variables  $t_1$  and  $t_2$  to walk along “paths”
- Assignments followed (not preceded) by concretization steps
- Check if in all concretizations  $t_1 = t_2$

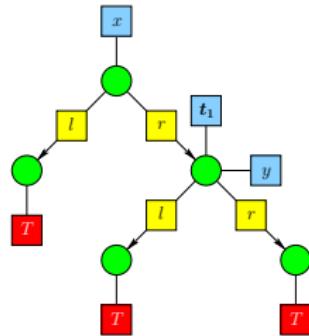
Does ASAT $\llbracket x.r.l.r = y.l.r, H \rrbracket = 1$  hold?



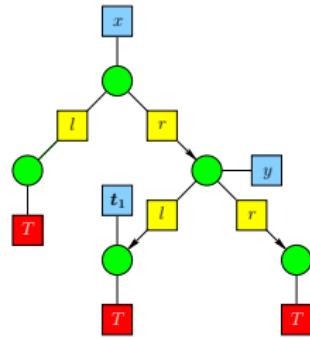
Does ASAT $\llbracket x.r.l.r = y.l.r, H \rrbracket = 1$  hold?

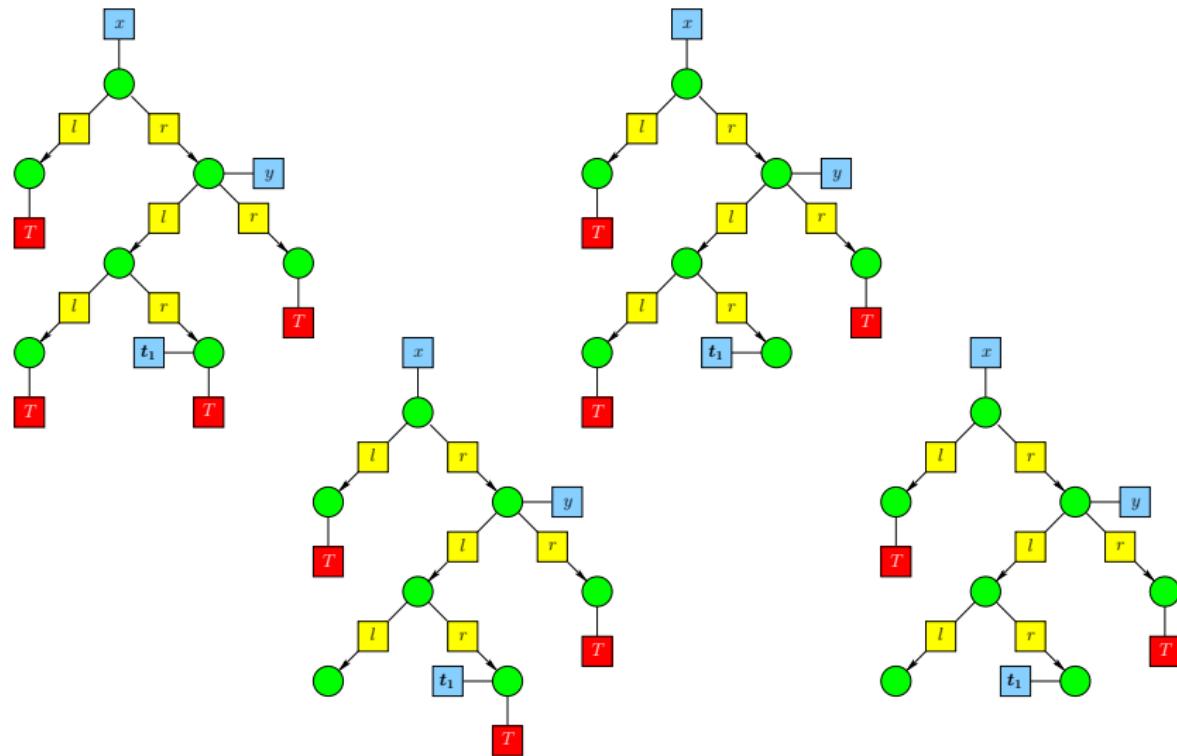


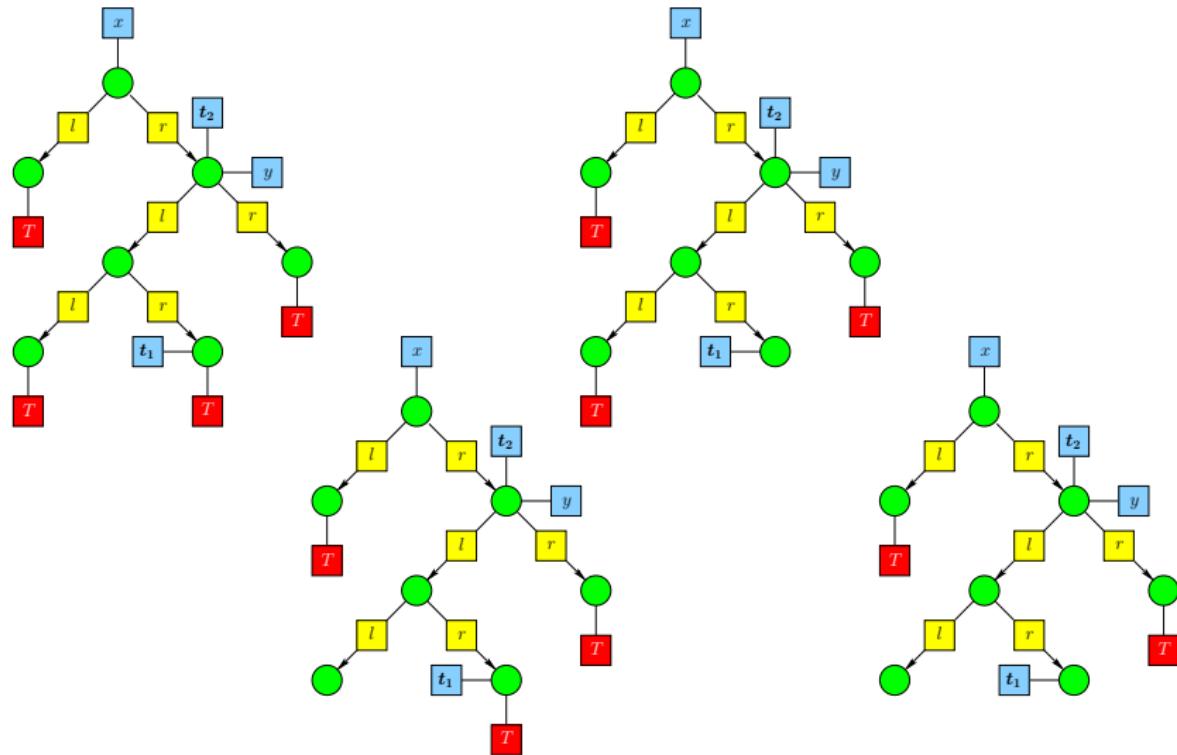
Does  $\text{ASAT}[\![x.r.l.r = y.l.r, H]\!] = 1$  hold?

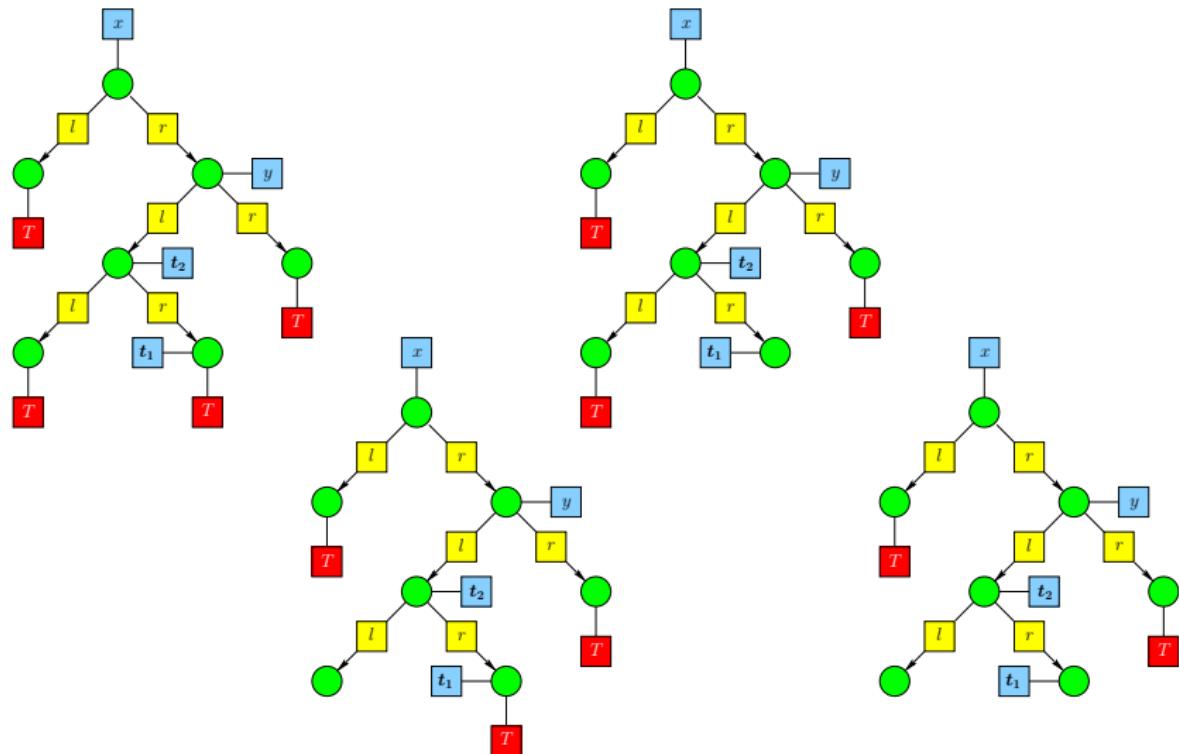


Does ASAT $\llbracket x.r.l.r = y.l.r, H \rrbracket = 1$  hold?

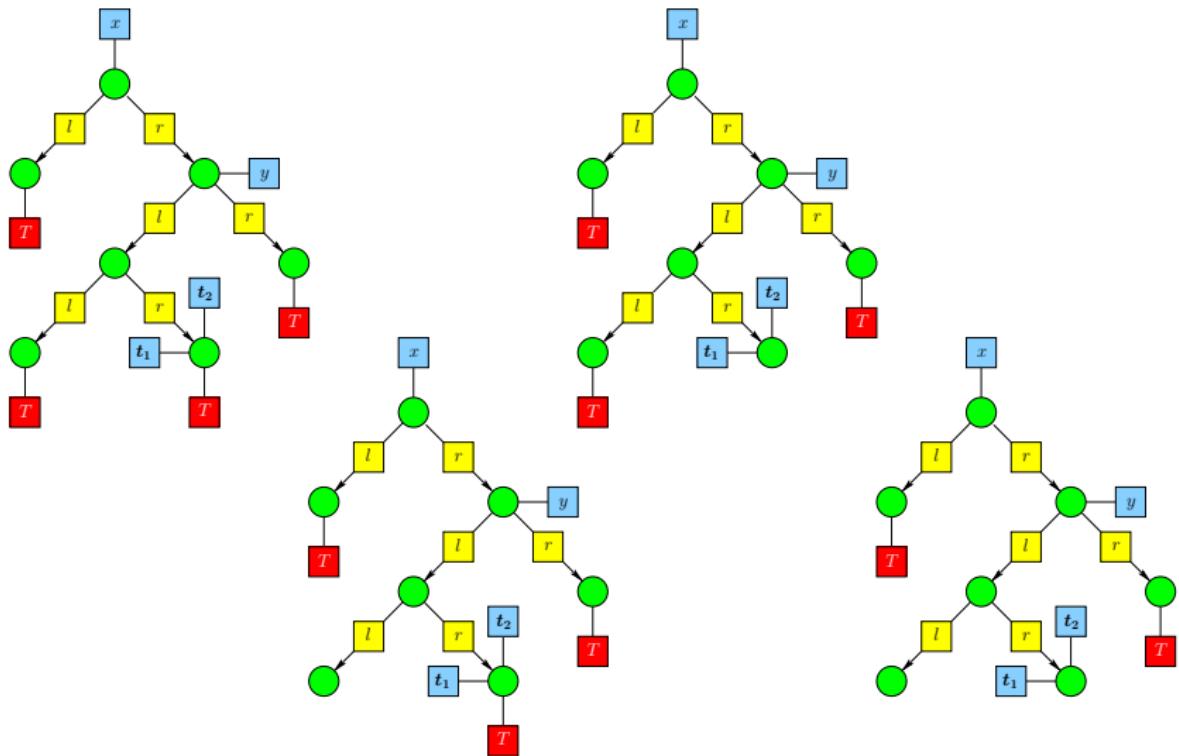


Does ASAT $\llbracket x.r.l.r = y.l.r, H \rrbracket = 1$  hold?

Does ASAT $\llbracket x.r.l.r = y.l.r, H \rrbracket = 1$  hold?

Does ASAT $\llbracket x.r.l.r = y.l.r, H \rrbracket = 1$  hold?

$$\forall K : \text{CSAT}[\![t_1 = t_2, H]\!] = 1 \Rightarrow \text{ASAT}[\![\dots]\!] = 1$$

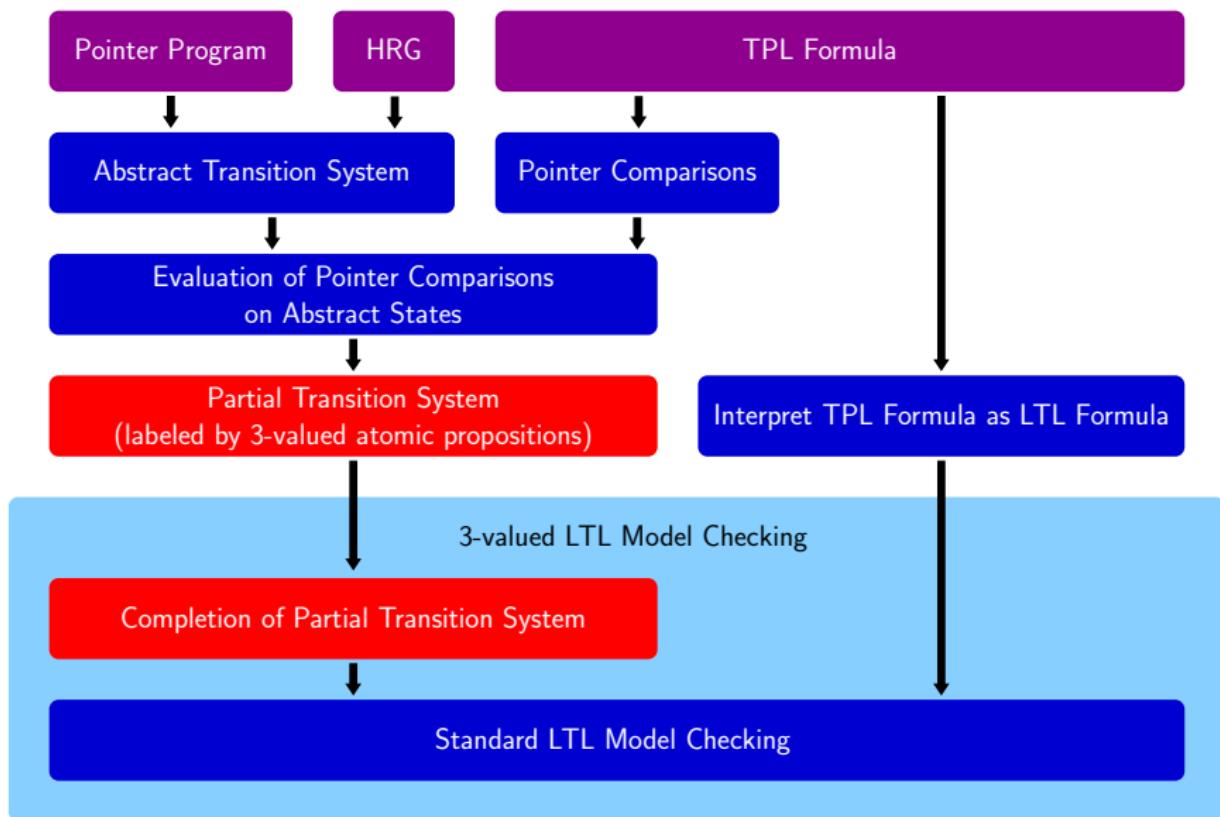


# A Special Case

## Limiting Dereferencing Depth

When dereferencing depth in pointer comparisons is limited to one, we **always** get clearly determined results (0 or 1).

**Reason:** admissibility of heap configurations



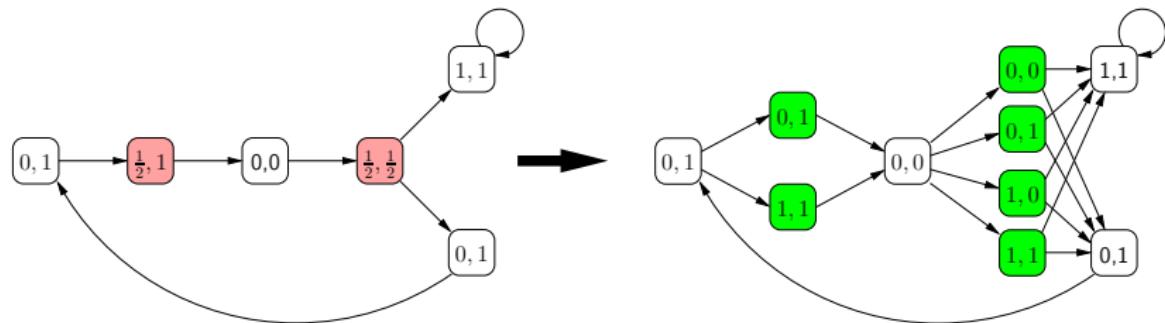
# Three-valued LTL Model Checking

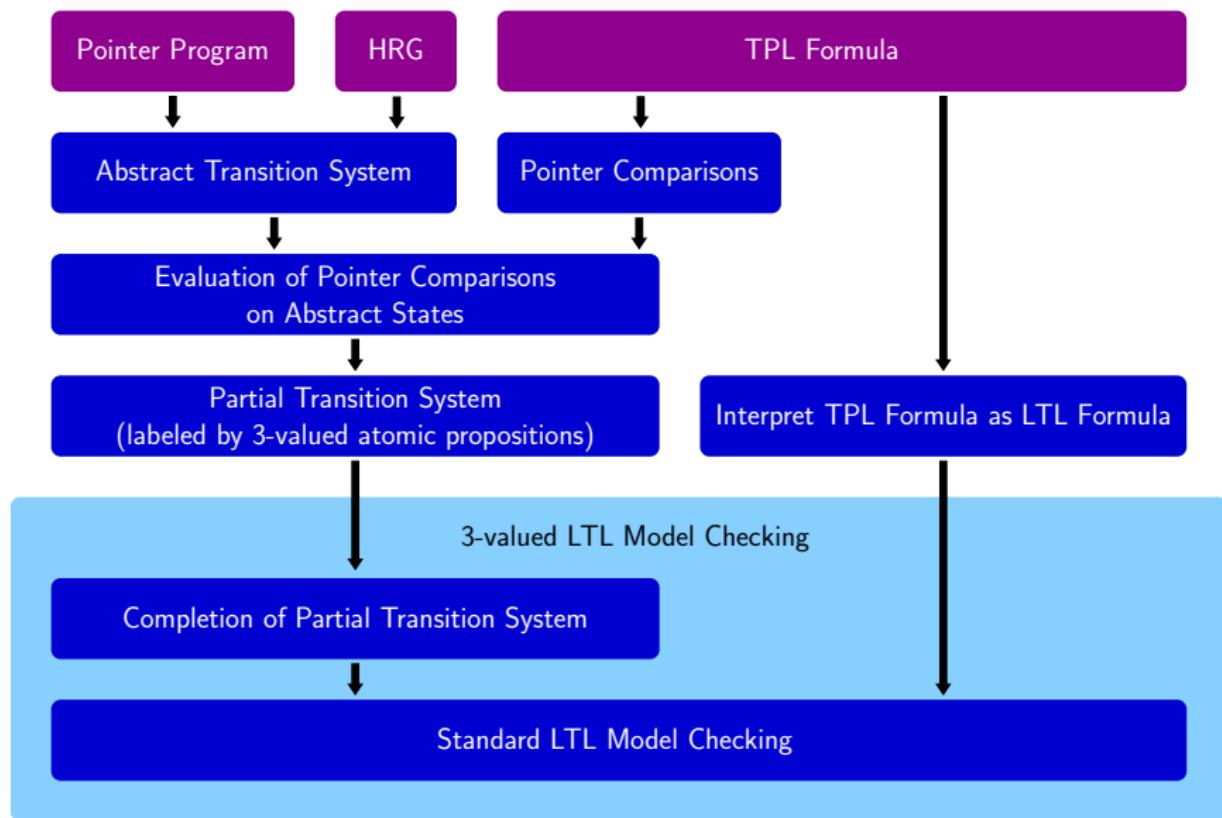
## Setting

- Evaluation of pointer comparisons can result in either  $0, 1$  or  $\frac{1}{2}$
- Transition system has **3-valued** labeling

## Transformation

Transform transition system to represent all possibilities for  $\frac{1}{2}$ -valued predicates.





# Quantifiers

## Quantified TPL

$$Q_1 X_1 Q_2 X_2 \dots Q_k X_k : \varphi(X_1, X_2, \dots, X_k)$$

- Quantification over heap objects present in the initial states
- Preservation of object identities between states by **nondeterministic marking** with variables
- For every quantor an additional marking is necessary (exponential blow-up of state space)

# Example: The Deutsch-Schorr-Waite Algorithm

**Pointer Safety:** No pointer errors / null dereferences

**Shape Safety:** Input structure is retained

**Completeness:** all vertices are visited at least once

$$\forall \textcolor{red}{X} : \neg (cur \neq \textcolor{red}{X} \text{ } \mathbf{U} \text{ } \text{term})$$

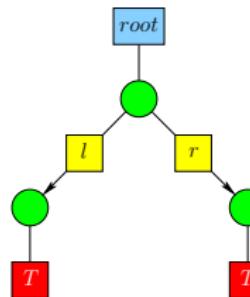
**Termination:** finally  $X$  never points to  $cur$  anymore

$$\forall \textcolor{red}{X} : \mathbf{FG}(cur \neq \textcolor{red}{X})$$

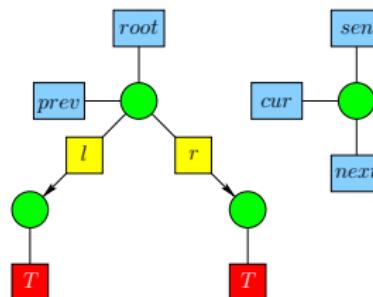
**Correctness:** for all vertices the left- and right successors are the same after program termination

$$\begin{aligned} \forall \textcolor{red}{X} \forall \textcolor{green}{X}_l \forall \textcolor{blue}{X}_r : \textcolor{red}{X}.l = \textcolor{green}{X}_l \wedge \textcolor{red}{X}.r = \textcolor{blue}{X}_r \rightarrow \\ ((\textcolor{red}{X} = \text{root} \rightarrow \mathbf{G}(\textcolor{red}{X} = \text{root})) \\ \wedge \mathbf{G}(\text{term} \rightarrow (\textcolor{red}{X}.l = \textcolor{green}{X}_l \wedge \textcolor{red}{X}.r = \textcolor{blue}{X}_r))) \end{aligned}$$

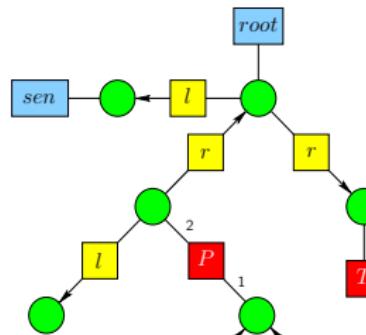
# Experimental Results: Verifying the DSW Algorithm



an initial heap



a final heap



an intermediate heap state

## Experimental Results: Verifying the DSW Algorithm

	no marking	1 marking	3 markings	TVLA
Initial States	5	185	962	
Number of States	20,678	6,220,798	35,983,627	> 80,000
Number of Transitions	23,359	7,078,257	40,909,648	
State Space Gen. (h:min:sec)	<0:01	10:14	1:18:03	
Memory Consumption	41 MB	788 MB	3,900 MB	150 MB
Pointer Safety	on-the-fly	-	-	
Shape Safety	on-the-fly	-	-	
Completeness (min:sec)	-	0:16	-	
Termination (min:sec)	-	0:39	-	
Correctness (min:sec)	-	-	4:05	
Total Time (State Space Gen. + all Properties)			1:28:35	<9:00:00

## Conclusion

- analysis and verification of **complex** data structures
- **highly parametrized** framework
- **handling of inconsistencies** wrt. the data structure
- more **intuitive** than other approaches
- promising experimental results

## Conclusion

- analysis and verification of **complex** data structures
- **highly parametrized** framework
- **handling of inconsistencies** wrt. the data structure
- more **intuitive** than other approaches
- promising experimental results

## Additional Features

- abstraction-only grammars
- optimized concretization possible
- unbounded thread creation [Noll and Rieger, 2008]

## Conclusion

- analysis and verification of **complex** data structures
- **highly parametrized** framework
- **handling of inconsistencies** wrt. the data structure
- more **intuitive** than other approaches
- promising experimental results

## Additional Features

- abstraction-only grammars
- optimized concretization possible
- unbounded thread creation [Noll and Rieger, 2008]

## Outlook

- **learning** of HRGs
- **typed/attributed** HRGs

(FM 2008) Thomas Noll and Stefan Rieger. **Verifying Dynamic Pointer-Manipulating Threads**

(ICGT 2008) Stefan Rieger and Thomas Noll. **Abstracting Complex Data Structures by Hyperedge Replacement**

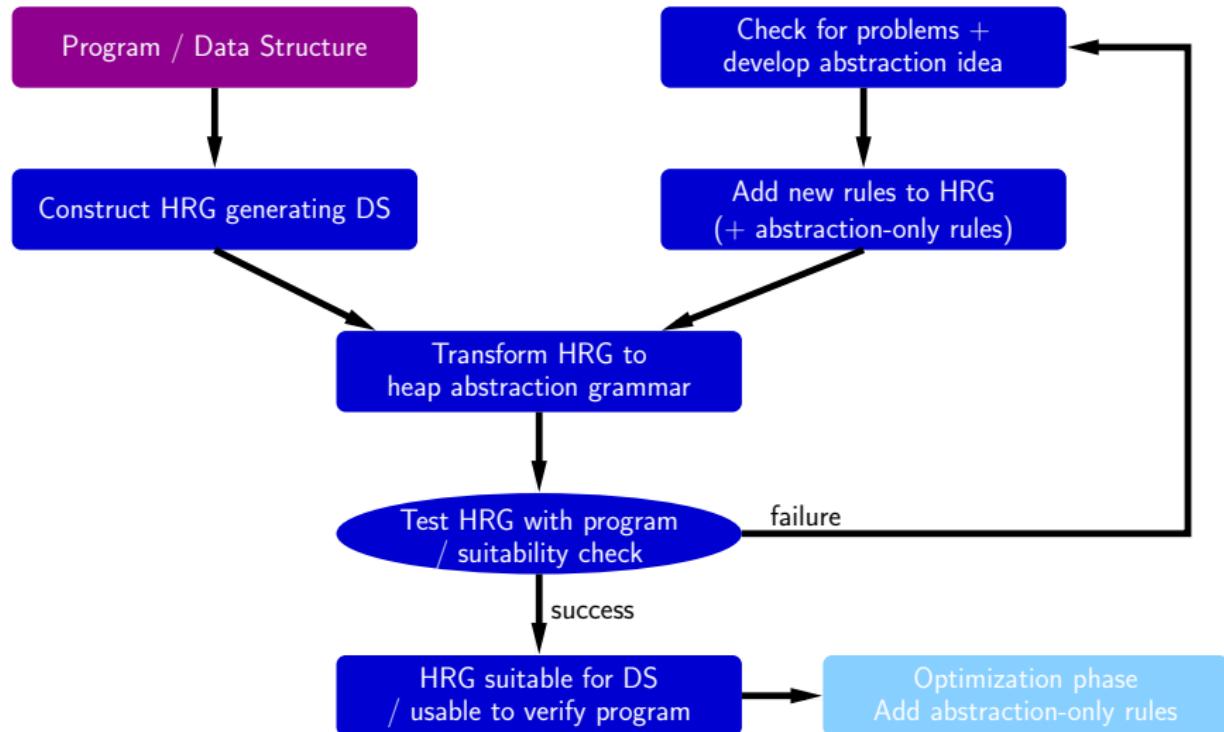
(TTSS 2009) Jonathan Heinen, Thomas Noll, and Stefan Rieger. **Juggrnaut: Graph Grammar Abstraction for Unbounded Heap Structures (to be published)**

(ICTAC 2007) Thomas Noll and Stefan Rieger. **Composing Transformations to Optimize Linear Code**

Thank you for your attention!

- (FM 2008) Thomas Noll and Stefan Rieger. **Verifying Dynamic Pointer-Manipulating Threads**
- (ICGT 2008) Stefan Rieger and Thomas Noll. **Abstracting Complex Data Structures by Hyperedge Replacement**
- (TTSS 2009) Jonathan Heinen, Thomas Noll, and Stefan Rieger. **Juggrnaut: Graph Grammar Abstraction for Unbounded Heap Structures (to be published)**
- (ICTAC 2007) Thomas Noll and Stefan Rieger. **Composing Transformations to Optimize Linear Code**

# Development of HRGs



# Partial Concretization II

## Solving the Problem

- Enforcing HRGs to be in **apex form** (for all  $X \rightarrow H$ , the nodes  $\text{ext}_H$  are only adjacent to terminals)

# Partial Concretization II

## Solving the Problem

- Enforcing HRGs to be in **apex form** (for all  $X \rightarrow H$ , the nodes  $\text{ext}_H$  are only adjacent to terminals)  
⇒ **impractical** [Engelfriet, 1992]

# Partial Concretization II

## Solving the Problem

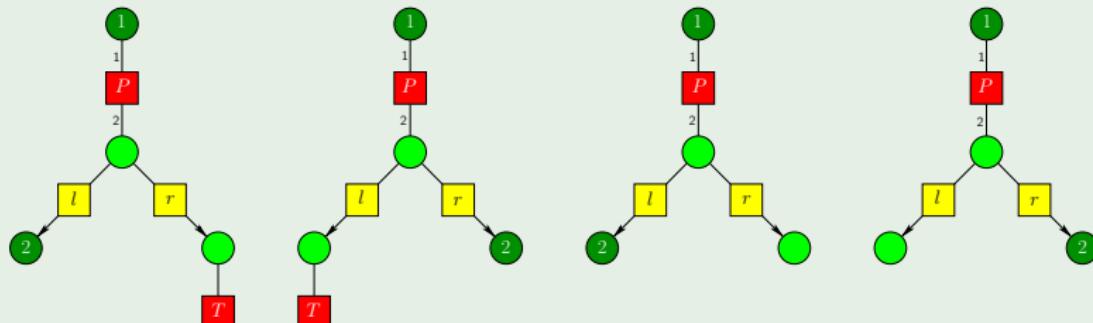
- Enforcing HRGs to be in **apex form** (for all  $X \rightarrow H$ , the nodes  $\text{ext}_H$  are only adjacent to terminals)
  - ⇒ **impractical** [Engelfriet, 1992]
  - ⇒ Introducing **additional redundant** grammar-rules that do not modify the language

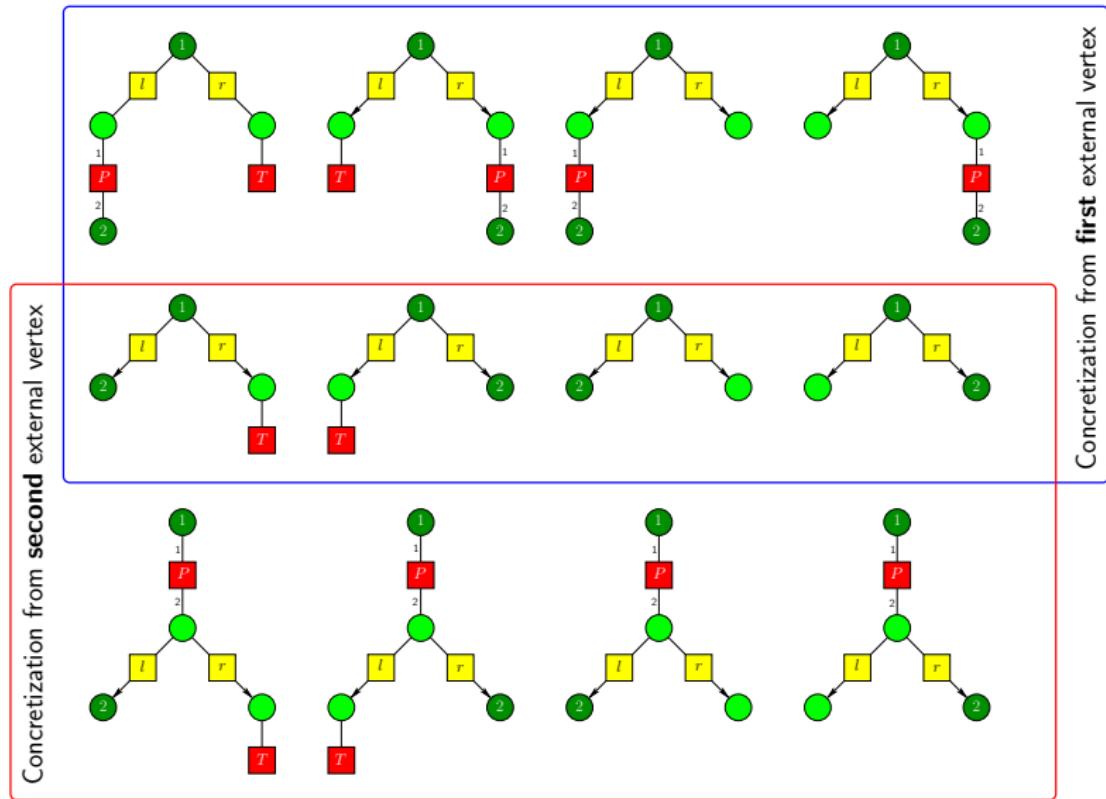
# Partial Concretization II

## Solving the Problem

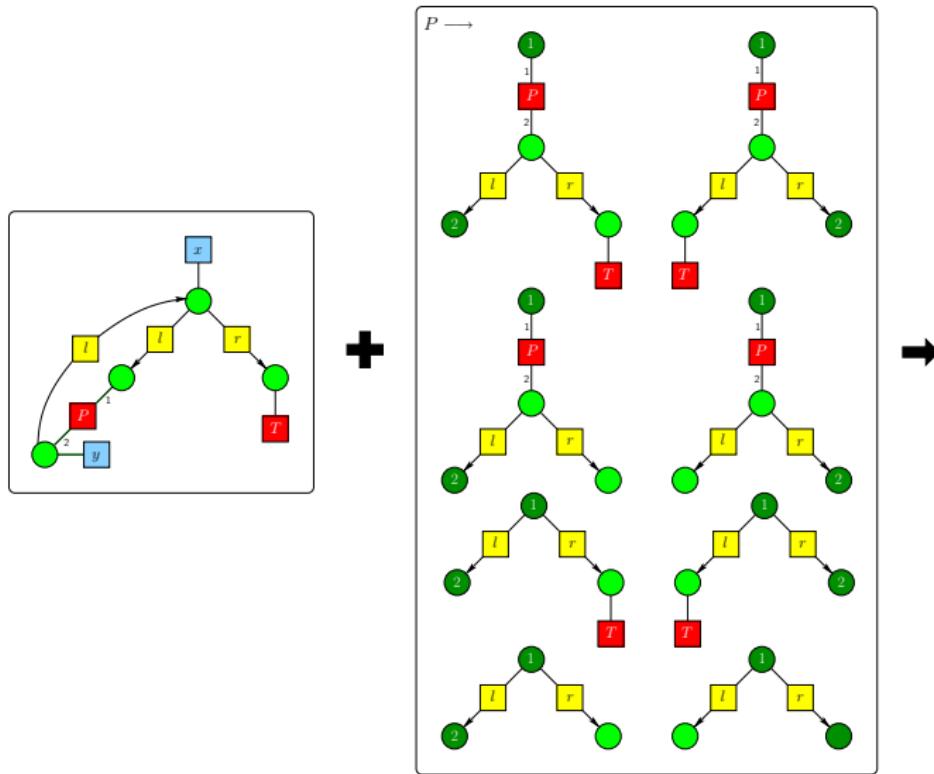
- Enforcing HRGs to be in **apex form** (for all  $X \rightarrow H$ , the nodes  $\text{ext}_H$  are only adjacent to terminals)
- ⇒ **impractical** [Engelfriet, 1992]
- ⇒ Introducing **additional redundant** grammar-rules that do not modify the language

## Additional Rules

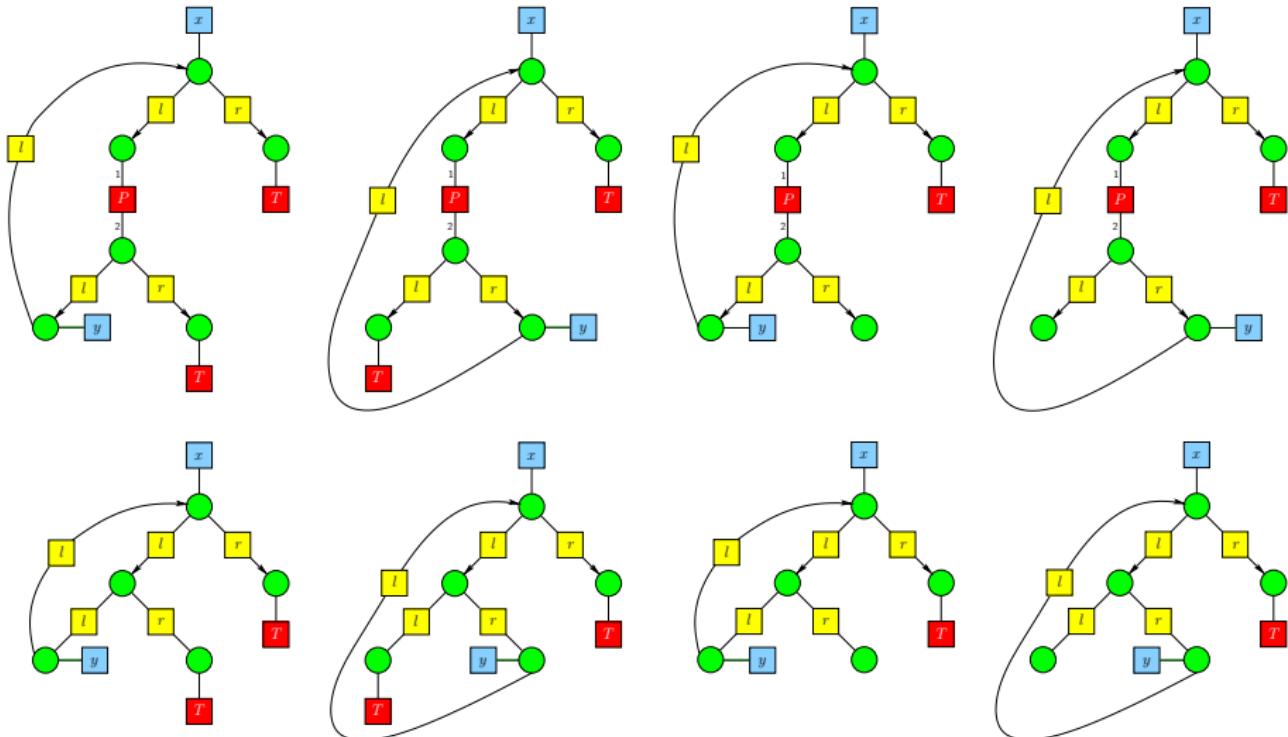


All  $P$ -Rules

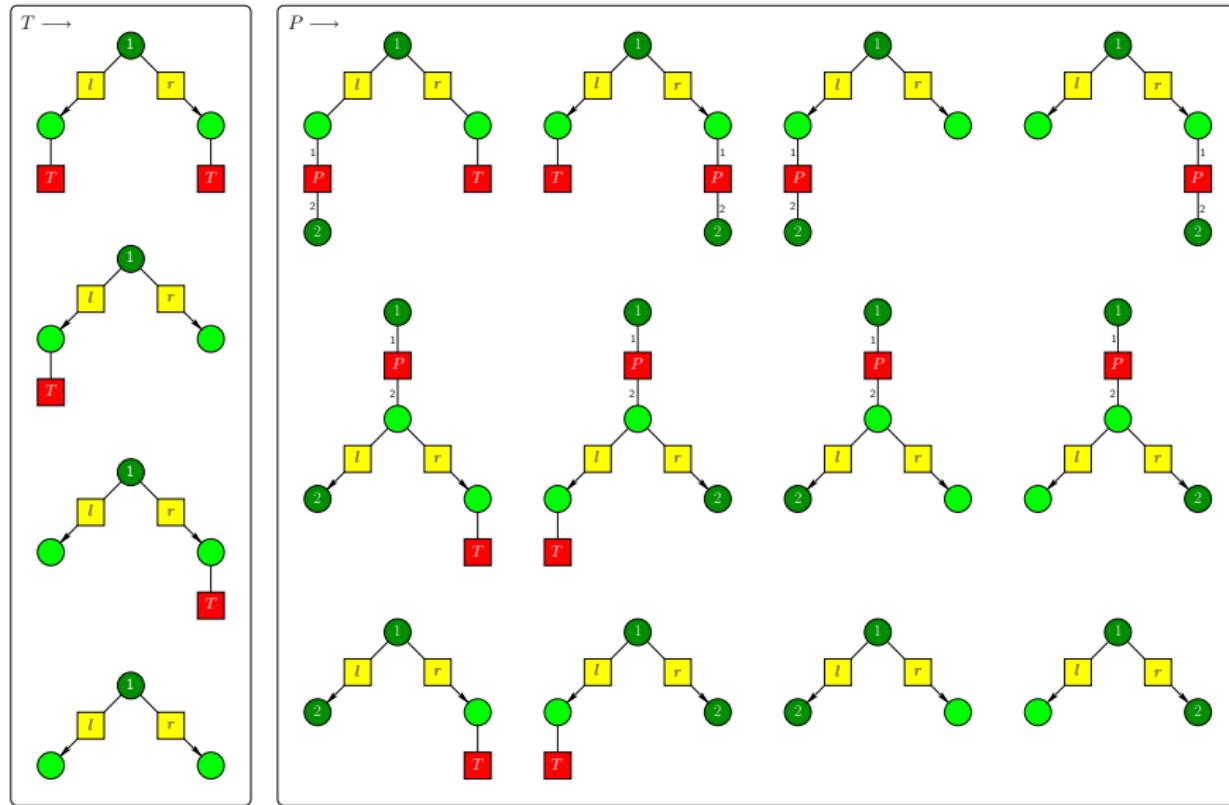
## Again the Example



## Admissible Results



## Entire Grammar



-  Bakewell, A., Plump, D., and Runciman, C. (2004a).  
Checking the shape safety of pointer manipulations.  
In *Relational and Kleene-Algebraic Methods in Computer Science '03*, volume 3051 of *Lecture Notes in Computer Science*, pages 48–61. Springer.
-  Bakewell, A., Plump, D., and Runciman, C. (2004b).  
Specifying pointer structures by graph reduction.  
In *Applications of Graph Transformations with Industrial Relevance '03*, volume 3062 of *Lecture Notes in Computer Science*, pages 30–44. Springer.
-  Baldan, P., Corradini, A., and König, B. (2004).  
Verifying Finite-State Graph Grammars: An Unfolding-Based Approach.  
In *CONCUR '04*, volume 3170 of *Lecture Notes in Computer Science*, pages 83–98. Springer.
-  Baldan, P. and König, B. (2002).

Approximating the behaviour of graph transformation systems.  
In *1st International Conference on Graph Transformations, ICGT 2002*, volume 2505 of *Lecture Notes in Computer Science*, pages 14–29. Springer.

-  Beyer, D., Henzinger, T. A., and Théoduloz, G. (2006).  
Lazy shape analysis.  
In *Computer Aided Verification, 18th International Conference, CAV '06*, volume 4144 of *Lecture Notes in Computer Science*, pages 532–546. Springer.
-  Dodds, M. and Plump, D. (2006).  
Extending C for checking shape safety.  
In *Graph Transformation for Verification and Concurrency '05*, volume 154(2) of *ENTCS*, pages 95–112. Elsevier.
-  Engelfriet, J. (1992).  
A Greibach Normal Form for Context-Free Graph Grammars.

In *19th International Colloquium on Automata, Languages and Programming, ICALP 1992*, volume 623 of *Lecture Notes in Computer Science*, pages 138–149. Springer.

-  Kastenberg, H. and Rensink, A. (2006).  
Model checking dynamic states in GROOVE.  
In *Model Checking Software (SPIN '06)*, volume 3925 of *Lecture Notes in Computer Science*, pages 299–305. Springer.
-  Noll, T. and Rieger, S. (2008).  
Verifying dynamic pointer-manipulating threads.  
In *15th International Symposium on Formal Methods (FM '08)*, volume 5014 of *Lecture Notes in Computer Science*, pages 84–99. Springer.
-  O'Hearn, P. W., Yang, H., and Reynolds, J. C. (2004).  
Separation and information hiding.

In *Proceedings of the 31st ACM SIGPLAN-SIGACT Symposium on Principles of Programming Languages, POPL 2004*, pages 268–280. ACM Press.

-  Rensink, A. (2004).  
Canonical graph shapes.  
In *Proc. of 13th European Symposium on Programming (ESOP '04)*, volume 2986 of *Lecture Notes in Computer Science*, pages 401–415. Springer.
-  Rensink, A. and Distefano, D. (2006).  
Abstract graph transformation.  
In *Proc. of Int. Workshop on Software Verification and Validation (SVV '05)*, volume 157(1) of *Electr. Notes Theor. Comput. Sci.*
-  Reynolds, J. C. (2002).  
Separation logic: A logic for shared mutable data structures.

In *IEEE Symposium on Logic in Computer Science, LICS 2002*,  
pages 55–74. IEEE Computer Society.



Sagiv, M., Reps, T., and Wilhelm, R. (2002).  
Parametric shape analysis via 3-valued logic.  
*ACM Transactions on Programming Languages and Systems*,  
24(3):217–298.