Testing of Reactive Systems

Lecture 2: Modelling Reactive Systems

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Lehrstuhl Informatik 2 (MOVES)
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1. Actions
2. Labelled Transition Systems
3. A language to describe LTS

Actions

Actions are

- express activity of the modelled system
- are executed
- are atomic (execution is indivisible)

Actions are used to observe or to influence a system.
Actions might be

- triggered
- or inhibited

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Example

Triggered or Inhibited

Consider action $a \hat{=}$ "Receiving a message over some channel"

Triggered: a triggered, if somebody actually sends a message over the channel

Inhibited: a inhibited, if there is no message

Actions

Definition

Let *Act* be the set of actions.

Note

Actions similar to symbols in an Alphabet (*cf.* Automata Theory).

- Act^*: the set of finite words over Act
- ε: the empty word
- $Act^+ = Act^* \setminus \varepsilon$
- $v \cdot w$: concatenation of words $v, w \in Act^*$

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Observability

- Actions $a \in Act$ are considered observable
- Let $\tau \notin Act$: τ is the
  - silent or
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Why only one unobservable action?

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$Act_\tau = Act \cup \{\tau\}$.

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Labelled Transition Systems

Labelled Transition System

- One of the most fundamental models in theoretical computer science
- Ingredients: States, Transitions, Actions

Definition

A Labelled Transition System L is a tuple $L = (S, Act, \rightarrow)$, with:

- S is a set of states
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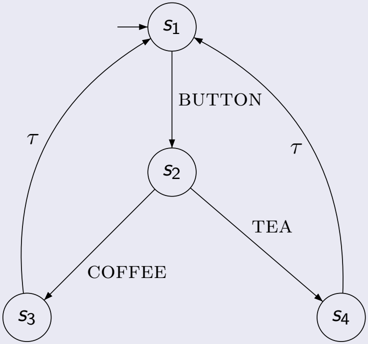
Example

Coffee 1

Let $L = (S, Act, \rightarrow)$
$S = \{s_1, s_2, s_3, s_4\}$
$Act = \{\text{COFFEE}, \text{TEA}, \text{BUTTON}\}$.

- $(s, a, s') \in \rightarrow$: s source state, s' target state
- We write $s \xrightarrow{a} s'$ if $(s, a, s') \in \rightarrow$.



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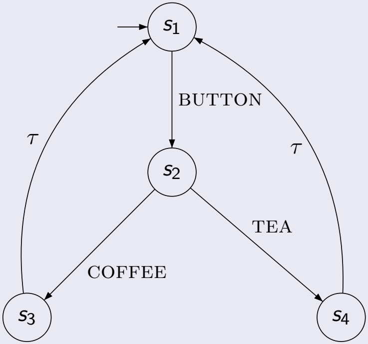

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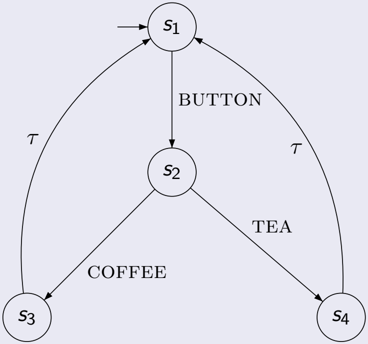

Deadlock

Deadlock

- $s \in S$ absorbing: $s \stackrel{a}{\not\rightarrow} \forall a \in Act_\tau$
- Alternatively: s is deadlocked
- Abbreviation: $s \not\rightarrow$ iff s is deadlocked.

Derived Transition Relations

Definition: Generalising $\rightarrow$

For $\sigma = a_1 \cdot a_2 \cdot \cdots \cdot a_n \in Act_\tau^+$:

$$p \xrightarrow{\sigma} p' \quad \text{iff} \quad \exists p_0, \ldots, p_n \in S : p_0 \xrightarrow{a_1} p_1, p_1 \xrightarrow{a_2} p_2, \cdots, p_{n-1} \xrightarrow{a_n} p_n$$

where $p = p_0$ and $p' = p_n$.

Note that τ is allowed in σ.

Abbreviations

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Definition: Abstracting from τ

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\begin{aligned}
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- With the $\Longrightarrow$ relation we abstract from τ transitions.
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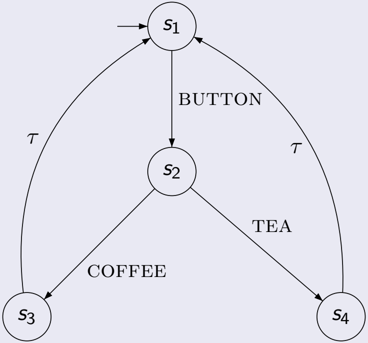
Some derived transitions



1. $s_1 \xrightarrow{\text{BUTTON}\cdot\text{COFFEE}} s_3$
2. $s_1 \xrightarrow{\text{BUTTON}\cdot\text{COFFEE}\cdot\tau} s_1$
3. $s_1 \overset{\text{BUTTON}\cdot\text{TEA}}{\Longrightarrow} s_4$, but also
4. $s_1 \overset{\text{BUTTON}\cdot\text{TEA}}{\Longrightarrow} s_1$

Traces

Describing Dynamic Behaviour of LTS

- $\exists$ many different approaches to describe behaviour of LTS
- most basic: traces

Definition: Traces

Let $s \in S$. The set of traces of s, denoted $traces(s)$, is defined as

$$traces(s) = \{\sigma \in Act^* \mid s \stackrel{\sigma}{\Longrightarrow}\}.$$

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- traces $traces(s)$ are actually a language
- if we see LTS L as an NFA with
  - start state s
  - all states $s' \in S$ accepting

  then $traces(s)$ is the language accepted by this automaton.

One more note

- We will refer to all words $\sigma \in Act^*$ as traces
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$$traces(s_3) = \{\varepsilon,$$
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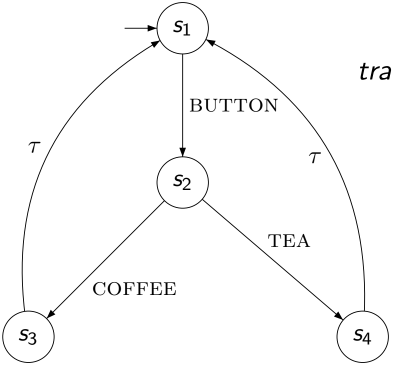
$$\ldots\} = traces(s_1) = traces(s_4)$$



$$traces(s_2) = \{\varepsilon,$$
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$$\ldots\}$$
$$= \text{COFFEE} \cdot traces(s_1)$$
$$\cup \text{TEA} \cdot traces(s_1)$$
$$\cup \{\varepsilon\}$$

Reachable states

What states can be reached from state s with trace σ?

Definition: after

For $s \in S, \sigma \in Act^*$:

- s **after** $\sigma := \{s' \mid s \overset{\sigma}{\Longrightarrow} s'\}$
- For $S' \subseteq S$: S **after** $\sigma := \bigcup_{s \in S'} s$ **after** σ
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s **after** Act^* are called the derivatives of s, or reachable states from s.

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Nondeterminism

Definition: Deterministic LTS

A state $s \in S$ is called deterministic iff

$$\forall \sigma \in traces(s) : |\underline{s \textbf{ after } \sigma}| = 1$$

- An LTS is called determinstic, if all its states are deterministic
- An LTS that is not deterministic is non-deterministic

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Equivalent is: $\forall \sigma \in Act^* : |\underline{s \textbf{ after } \sigma}| \leq 1$ (Why?).

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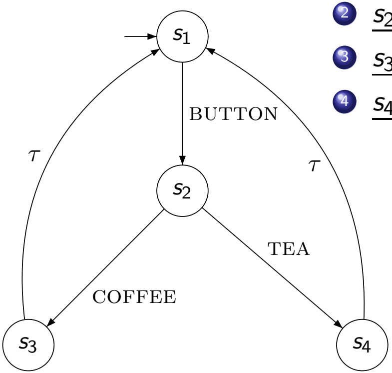
Nondeterminism

Sources of nondeterminism

1. nondeterministic branching: two outgoing transitions with same action
2. τ-transitions

Example



1. s_1 **after** BUTTON · COFFEE =
2. s_2 **after** TEA =
3. s_3 **after** ε =
4. s_4 **after** BUTTON · TEA =

Nondeterminism is here solely caused by the τ-transitions.

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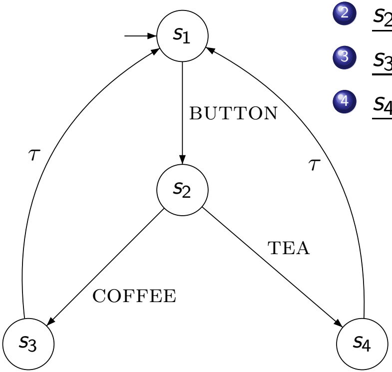

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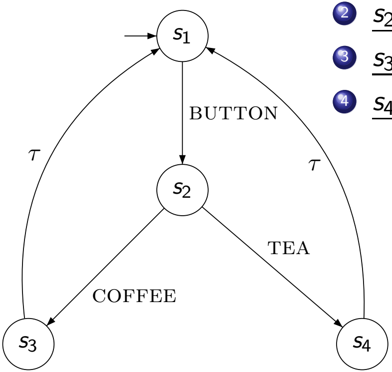

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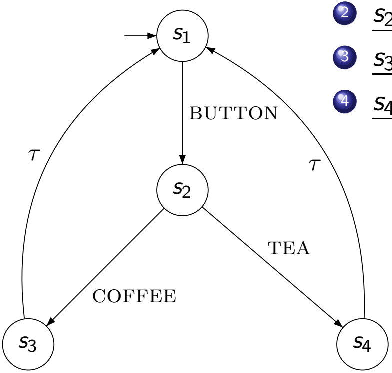

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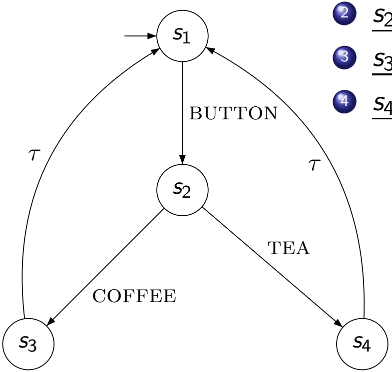

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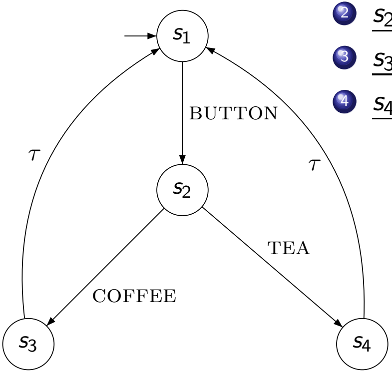

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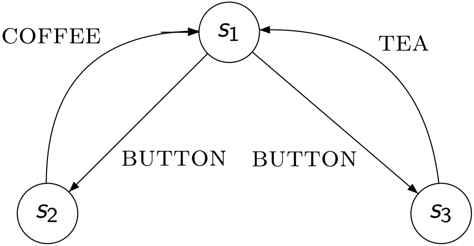

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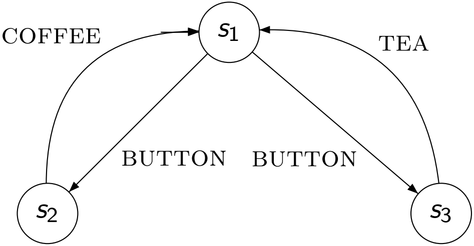

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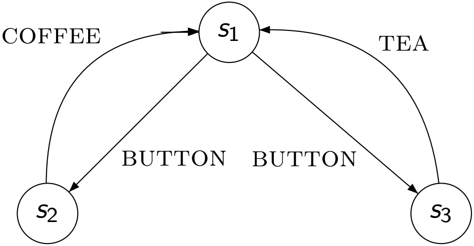

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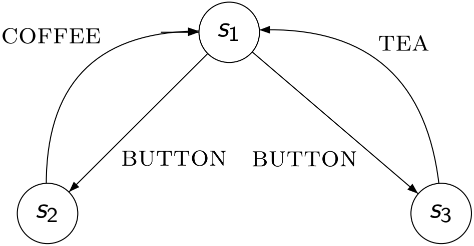

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A language to describe LTS

Reminder

Known from automata theory: regular expressions.

- 0 is a regular expression.
- 1 is a regular expression.
- for $a \in Act$: a is a regular expression.
- for e, e' regular expressions:
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Regular expressions can be turned into finite automata

We shall now define expressions that describe LTS. We will call these expressions processes.

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- 1 is a regular expression.
- for $a \in Act$: a is a regular expression.
- for e, e' regular expressions:
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Regular expressions can be turned into finite automata

We shall now define expressions that describe LTS. We will call these expressions processes.

A language to describe LTS

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A language to describe LTS

Definition: Processes $\mathbb{P}$

- Let $\mathcal{P}$ be the set of process variables.
- Let Act be a set of actions.
- The set $\mathbb{P}$ of processes is the language defined by the following grammar:

$$p \quad \rightarrow \quad \mathrm{STOP} \;\Big|\; a.p \;\Big|\; p+p \;\Big|\; p\|_A p \;\Big|\; P$$

where $a \in Act_\tau$, $A \subseteq Act$, and $P \in \mathcal{P}$.

- Process definitions are of the form

$$P \quad \triangleq \quad p$$

with $p \in \mathbb{P}$ and $P \in \mathcal{P}$.

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$$p \quad \rightarrow \quad \mathrm{STOP} \;\Big|\; a.p \;\Big|\; p{+}p \;\Big|\; p\|_A p \;\Big|\; P$$

Informal Meaning

Let $p, q \in \mathbb{P}$.

STOP: is the process that does nothing, is deadlocked.

$a.p$: executes action $a \in Act_\tau$ and behaves like process p.
the prefix operator

$p + q$: behaves either like process p or q.
the choice operator

$p\|_A q$: behaves like p and q running in parallel, synchronising over synchronisation set A
the parallel operator

P: if $P \triangleq p$, then P behaves exactly like p

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Note

The parallel operator makes this language very powerful:

- componentwise independent specification possible
- combination by parallel composition

Note 2

- Behaviour of a process can be described by LTS.
- processes are also states, *i.e.*, $\mathbb{P}$ is the set of states of the LTS that we will consider.

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Example 1.3.2: Some simple processes

A language to describe LTS

Recursion

- Up til now only terminating (= deadlocking) processes
- Use process definitions for non-terminating behaviour
- . . . recursive process definitions

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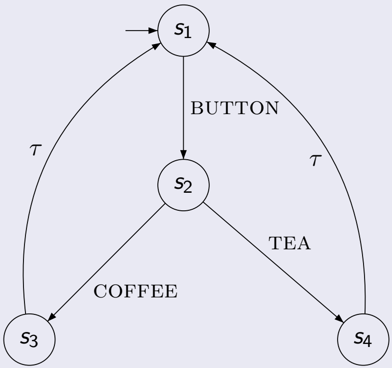
Example 1.3.3

Coffee 1



Example 1.3.3

Coffee 2