Satisfiability Checking

Prof. Dr. Erika Ábrahám

Theory of Hybrid Systems
Informatik 2

WS 11/12

Organization

- Language: English or German
- Lecture (V3):

  Monday 15:30-17:00 Room 5056
  Thursday 10:00-10:45 Room 5056

  Registration in L^{2}P learning room via Campus required.

  All materials are available in the learning room.
- Exercise (Ü1):

  Thursday, 10:45-11:30 room 5056, after the lecture

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- Assistants: Florian Corzilius, Nils Jansen, Ulrich Loup

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No lecture/exercise this Thursday!

Structure of the lecture

Problem we want to solve: develop algorithms for the automated check of the satisfiability (or validity) of formulae in different logics.

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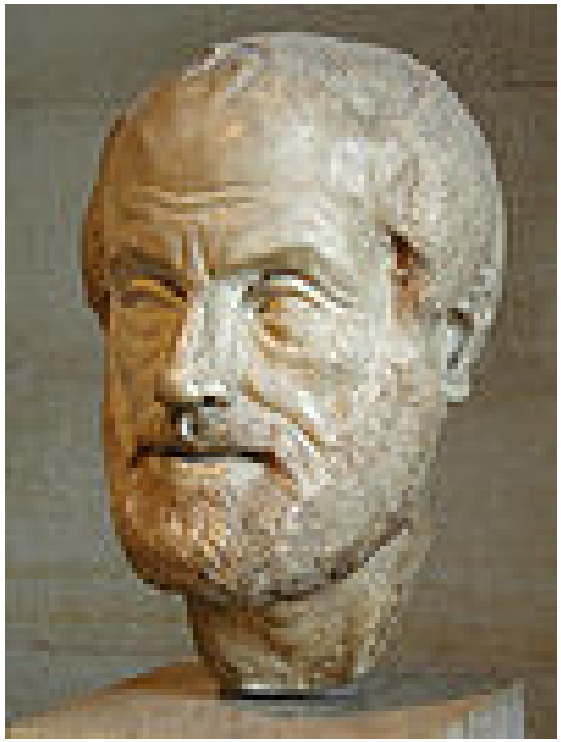
Philosophical logic

- 500 B.C - 19th century
- Logic dealing with sentences in the natural language used by humans.
- Example
  - All men are mortal.
  - Socrates is a man.
  - Therefore, Socrates is mortal.

Philosophical logic

- Natural languages are very ambiguous.
- Aristotle (384 BC – 322 BC) identified 13 types of fallacies in his *Sophistical Refutations*.



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  Implies that ours is best, or that ours is so poor that having none is the better choice.

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1. Human cells are invisible to the naked eye.
2. Humans are made up of human cells.
3. Therefore, humans are invisible to the naked eye.

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> Famously and controversially, in the Greek philosophy it was assumed that the atoms constituting a substance must themselves have the properties of that substance: so atoms of water would be wet, atoms of iron would be hard, atoms of wool would be soft, etc.

Fallacies (4)

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I had butterflies in my stomach.

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1. If I have the flu, then I have a sore throat.
2. I have a sore throat.
3. Therefore, I have the flu.

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> I think that we should make the academic requirements stricter for students. I recommend that you support this because we are in a budget crisis and we do not want our salaries affected.

This kind of irrelevant conclusion (red herring) is a debating tactic that seeks to divert an opponent.

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More and more young people are attending high schools and colleges today than ever before. Yet the rate of youth crime is also much higher then in earier times. This makes it clear that these young people are being corrupted by their education.

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$\rightarrow$ The conjunction of two true sentences is not always true.

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Symbolic and mathematical logic

- 1854: George Boole introduced symbolic logic and the principles of what is now known as Boolean logic.
- 1879: Gottlob Frege created with his *Begriffsschrift* the basis of modern logic with the invention of quantifier notation.
- 1910-1913: Alfred North Whitehead and Bertrand Russell published *Principia Mathematica* on the foundations of mathematics, attempting to derive mathematical truths from axioms and inference rules in symbolic logic.
- 1931: Gödel's incompleteness theorems (we will deal with them later).

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- Propositional logic - the foundation of computers and circuits
- Databases - Query languages
- Programming languages (e.g. Prolog)
- Specification and verification
- ...

Logic in computer science

- Propositional logic
- First order logic
- Higher order logic
- Temporal logic
- ...

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Linear arithmetic	$2x + y > 0 \wedge x + y \leq 0$ $2x = 1$
Real algebra	$x^2 + 2xy + y^2 < 0$