Heap Abstraction by Graph Reduction with Graph Grammars

Jonathan Heinen Thomas Noll Stefan Rieger

MOVES: Software Modeling and Verification
RWTH Aachen University, Germany

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FR 6.2 Informatik, Universität des Saarlandes, Saarbrücken

Overview

1. Introduction
2. Hypergraph Reduction
3. Heap Configurations
4. Modeling Program Semantics
5. Conclusion

Aim: Abstraction of complex data structures for verification

Problem

- **complex data structures (record structures)**
- creation and destruction of objects at runtime
- destructive updates via pointers
- parallelism
- possibly infinite state space

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Approach

- graph grammars to model data structures
- reverse rule application (graph reduction)
- partial concretization
- finiteness ensured by special sink node

Hypergraphs

Definition (Labeled Hypergraph)

Given: Alphabet Σ, ranking function $rk : \Sigma \to \mathbb{N}$
Define: $H = (V, E, att, \ell, ext)$ where

- set of nodes V, set of edges E

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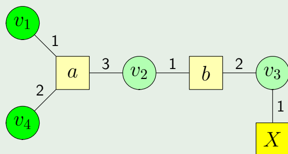
Example



$$
\begin{aligned}
V &= \{v_1, v_2, v_3, v_4\} \\
E &= \{e_1, e_2, e_3\} \\
att &= \{e_1 \mapsto v_1 v_4 v_2, e_2 \mapsto v_2 v_3, e_3 \mapsto v_3\} \\
\ell &= \{e_1 \mapsto a, e_2 \mapsto b, e_3 \mapsto X\} \\
ext &= v_1 v_4
\end{aligned}
$$

Hyperedge Replacement Grammar

Definition

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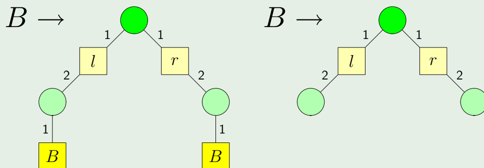
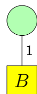
Example

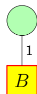
$$N = \{B\}$$
$$T = \{r, l\}$$
$$S = B$$



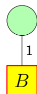
Derivation

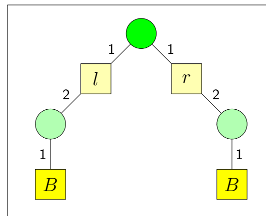


Derivation (Select Nonterminal)

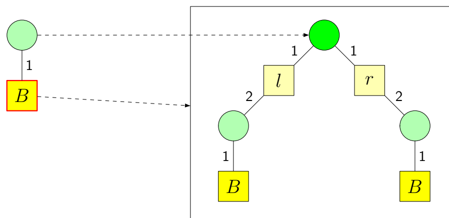


Derivation (Select Rule)

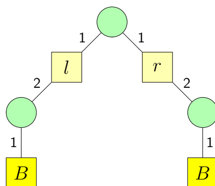




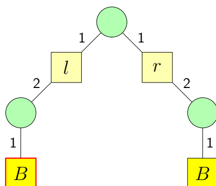
Derivation (Match)



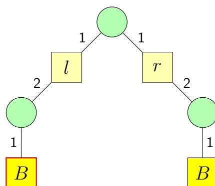
Derivation (Replace)

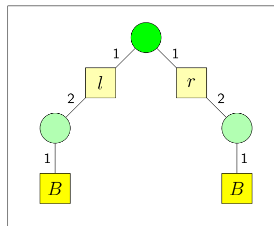


Derivation (Select Nonterminal)



Derivation (Select Rule)





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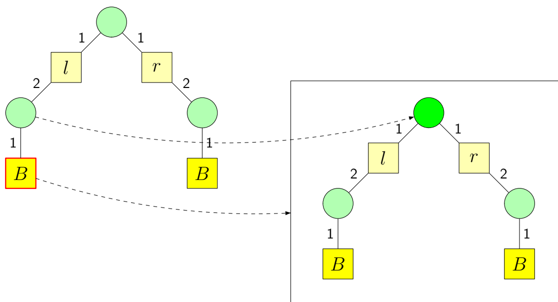
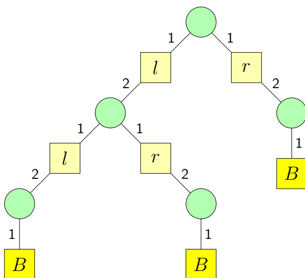


Derivation (Replace)



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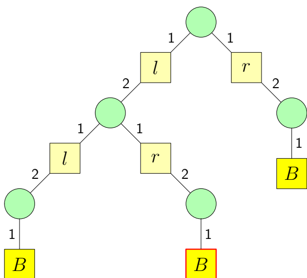

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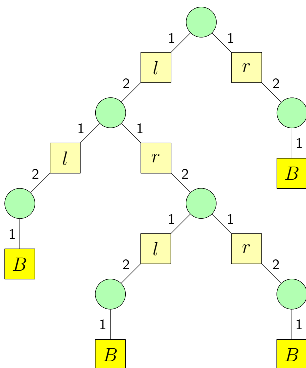

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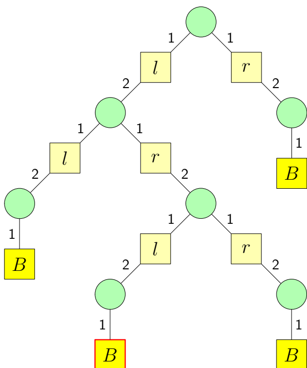
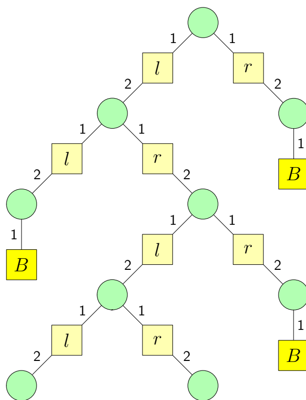
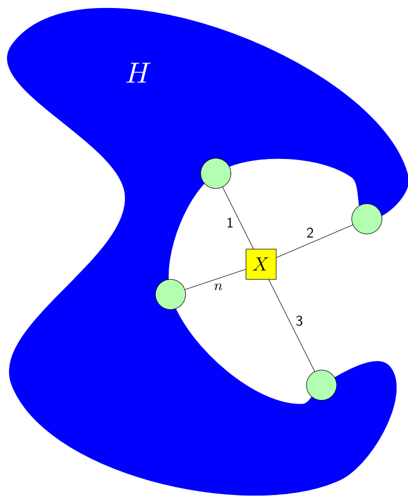


Derivation (Replace with Terminal Rule)



Rule Application (General Case)



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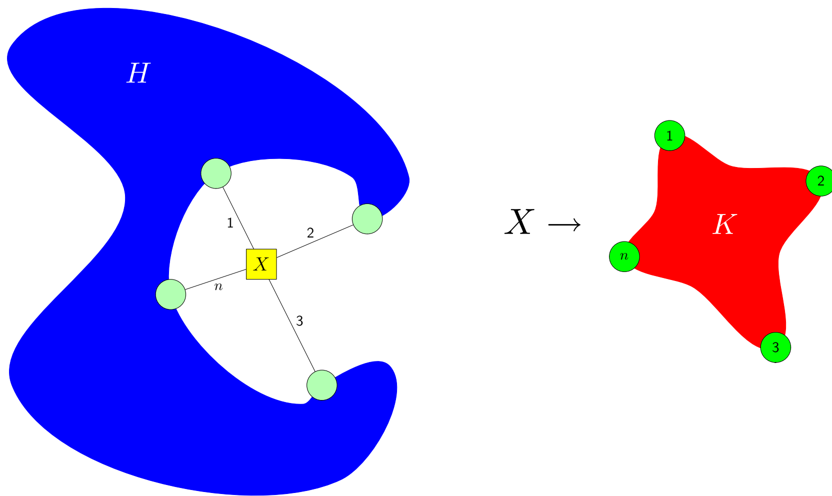

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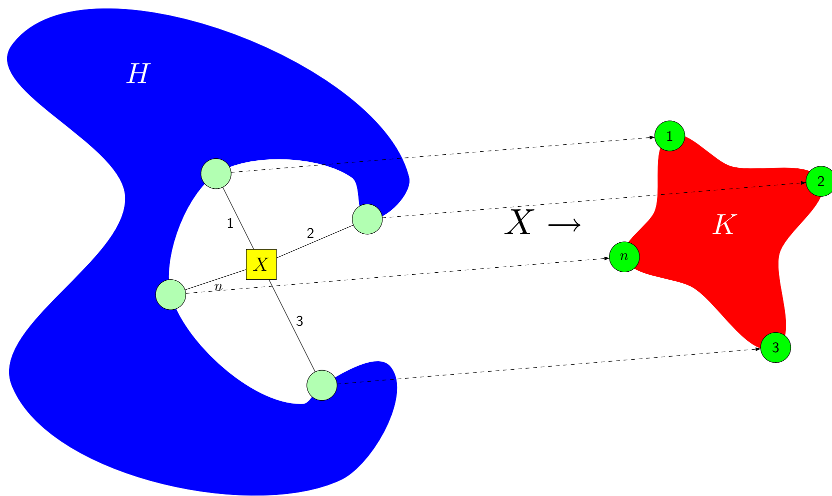

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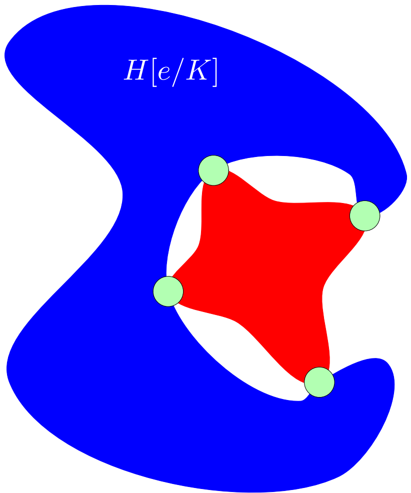

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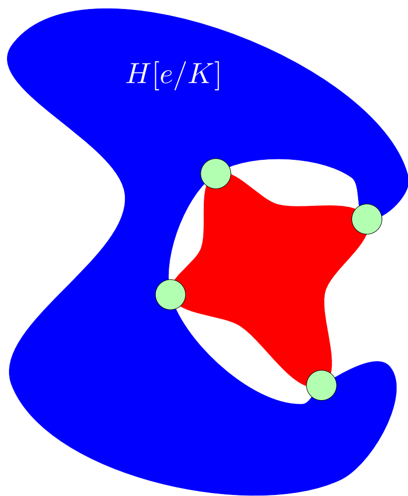

Some Properties

Sequentialization/Parallelization:

$H[e_1/H_1, ..., e_n/H_n] =$
$H[e_1/H_1]...[e_n/H_n]$

Confluence:

$H[e_1/H_1][e_2/H_2] =$
$H[e_2/H_2][e_1/H_1]$

Associativity:

$H[e_1/H_1][e_2/H_2] =$
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Hypergraph Reduction

Aim: Abstraction

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- Here: opposite approach, reduce size of graphs maintaining structural information

$\Rightarrow$ Backward application of grammar rules

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Problems

- Determining applicable rules
- Finding the redex
- No confluence, reduction might “get stuck” / different results
- No termination guaranteed (depending on grammar)

Hypergraph Embedding

Embedding $\iota : K \rightarrowtail H$

HG K is embedded in HG H if there is $\iota = (\iota_V, \iota_E)$ with

- $\iota_V : V_K \to V_H$ surjective on ext_K, bijective on $V_K \setminus [ext_K]$

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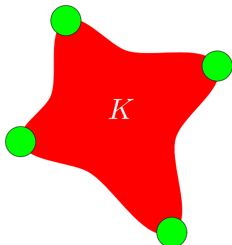
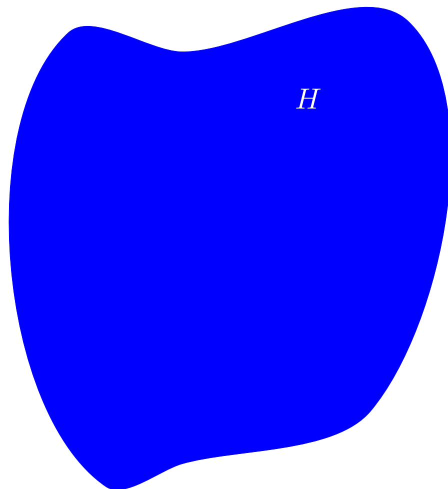
- $\iota_E : E_K \to E_H$ bijective
- functions retain ℓ and att
  (labeling & graph structure retained)

Backward Rule Application

$X \rightarrow$





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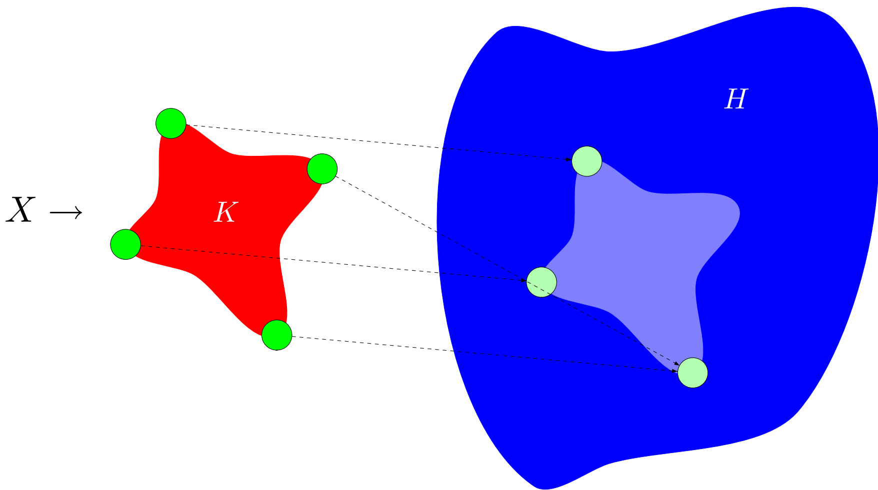
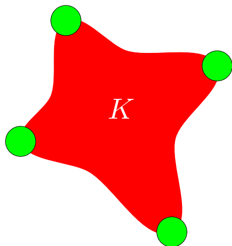
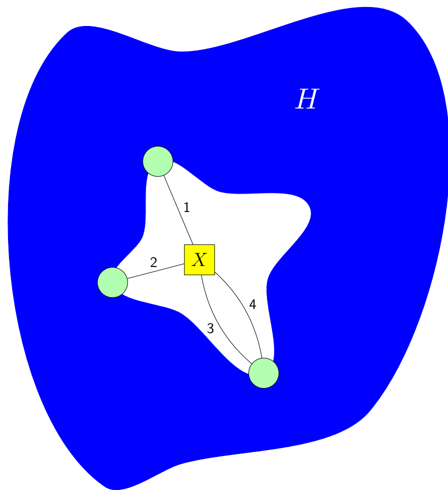


Backward Rule Application (Backward Replace)

$X \rightarrow$





Termination & Matching

Increasing HRG

A HRG $G = (N, T, P, S)$ is increasing iff for all $X \to H \in P$:

$$rk(X) < |V_H| \vee |E_H| > 1$$

Increasingness clearly implies finiteness of all backward derivations.

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How to match and select rules?

Define rule selector as partial function $\sigma(P, H) = (p, \iota)$ where $p = X \to K \in P$ and $\iota : K \rightarrowtail H$ such that $\exists H'$ with $H \xLeftarrow{p,\iota} H'$.

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Details? $\Rightarrow$ future work

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Heaps as Hypergraphs

Modeling

	Rank of Edges	Type of Label
pointers	2	terminal
program variables	1	variable (terminal)
abstract subgraphs	arbitrary	nonterminal

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Ensuring Finiteness

- Employ abstraction by (backward) application of HRG rules
- Enforce bound on the number of nodes

$\Rightarrow$ special sink node

Heap Configurations (formally)

Definition

Let G be a HRG. A heap configuration is of the form $\gamma = (H, Var, sink, F)$ with

- hypergraph H
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Boundedness

$\gamma = (H, Var, sink, F)$ is k–bounded if $|V_H| \leq k$.

Enforcing Boundedness

Idea

Summarize nodes in $sink$ if abstraction through HRG does not suffice for size reduction.

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Heap Compactor κ

$\kappa(\gamma, k) = \gamma'$ where $\gamma = (H, ...)$ and $\gamma' = (K, ...)$ s.t.:

- If $|V_H| > k : |V_K| = k$ and ...
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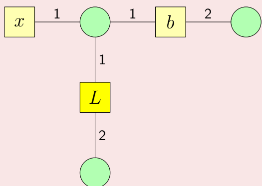
Strategies

- Summarize preferably connected nodes
- Summarize nodes as far away from variables as possible
- Call heap compactor always after abstraction

Partial Concretization

Problem: What is $x.a$?



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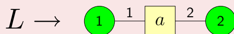
$L \longrightarrow$ 

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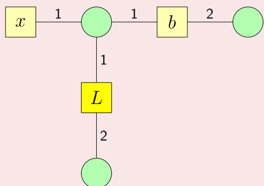
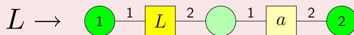
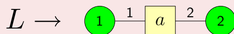


$L \longrightarrow$ 

$L \longrightarrow$ 

Idea: Apply rules nondeterministically until $x.a$ can be "resolved"
$\Rightarrow$ Possibly infite loop (analogous to left recursion by CFGs)

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Consequence

- Single rule application suffices
- Expressive power equivalent?

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Expressions and Assignments

Multivalued Expression Semantics

- pointer expressions yield a set of nodes, due to *sink* node
- boolean expressions cannot always be evaluated to a single value (due to pointer sem.)

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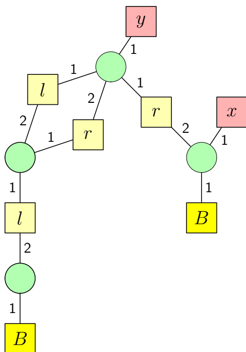
6. call to compactor to obtain k–bounded representation

Example

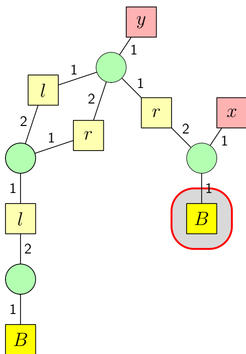


Instructions:

new(x.r);

Steps:

Example



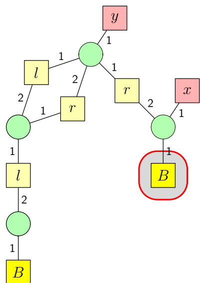
Instructions:

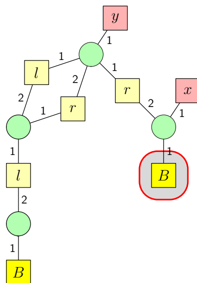
new(x.r);

Steps:

1. Heap Concretizer

Example





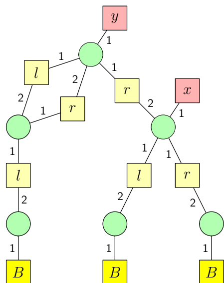
Instructions:

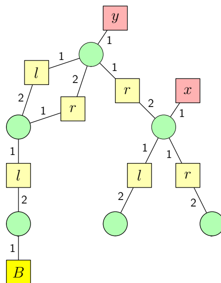
new(x.r);

Steps:

1. Heap Concretizer

Example





Instructions:

new(x.r);

Steps:

1. Heap Concretizer

Example

Instructions:

new(x.r);

Steps:

1. Heap Concretizer
2. Execution

Example

Instructions:

new(x.r);

Steps:

1. Heap Concretizer
2. Execution

Example

Instructions:

new(x.r);

Steps:

1. Heap Concretizer
2. Execution
3. Garbage Collector

Example

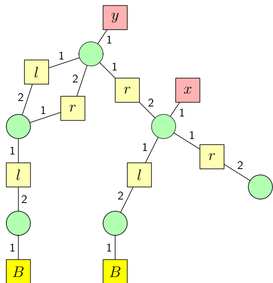
Instructions:

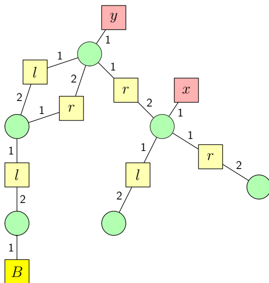
new(x.r);

Steps:

1. Heap Concretizer
2. Execution
3. Garbage Collector

Example





Instructions:

new(x.r);

Steps:

1. Heap Concretizer
2. Execution
3. Garbage Collector
4. Heap Abstractor

Example

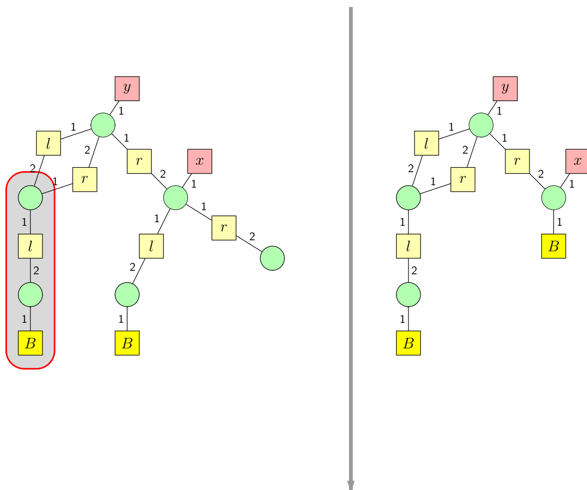
Instructions:

new(x.r);

Steps:

1. Heap Concretizer
2. Execution
3. Garbage Collector
4. Heap Abstractor

Example



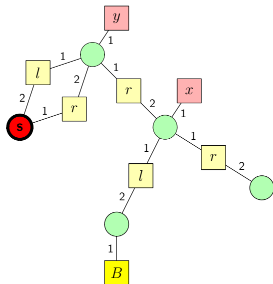
Instructions:

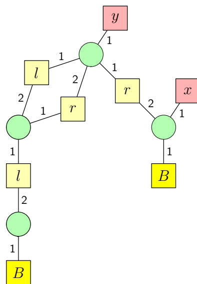
new(x.r);

Steps:

1. Heap Concretizer
2. Execution
3. Garbage Collector
4. Heap Abstractor
5. Heap Compactor

Example





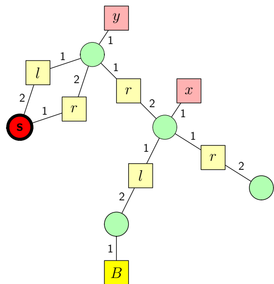
Instructions:

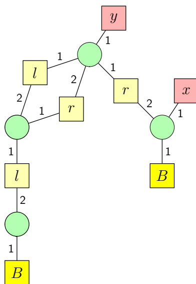
new(x.r);

Steps:

1. Heap Concretizer
2. Execution
3. Garbage Collector
4. Heap Abstractor
5. Heap Compactor

Example





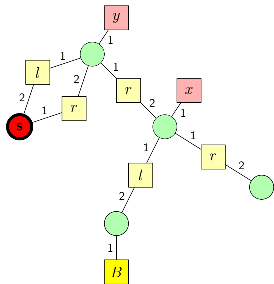
Instructions:

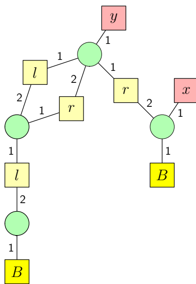
new(x.r);

x = y;

Steps:

Example





Instructions:

new(x.r);

x = y;

Steps:

1. Heap Concretizer

Example

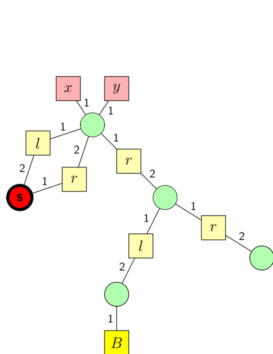
Instructions:

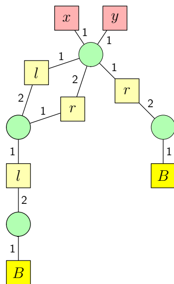
new(x.r);
x = y;

Steps:

1. Heap Concretizer
2. Execution

Example





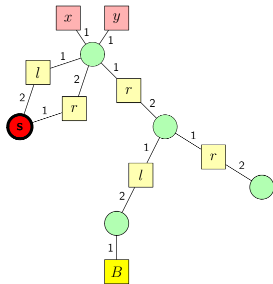
Instructions:

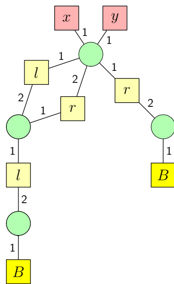
new(x.r);
x = y;

Steps:

1. Heap Concretizer
2. Execution

Example





Instructions:

new(x.r);
x = y;

Steps:

1. Heap Concretizer
2. Execution
3. Garbage Collector
4. Heap Abstractor
5. Heap Compactor

Overview

1. Introduction
2. Hypergraph Reduction
3. Heap Configurations
4. Modeling Program Semantics
5. Conclusion

Summary

- Abstraction framework using graph grammars
- Customization for many data structures possible (and necessary)
- Supports dynamic memory allocation, dynamic threading, destructive updates.

Summary

- Abstraction framework using graph grammars
- Customization for many data structures possible (and necessary)
- Supports dynamic memory allocation, dynamic threading, destructive updates.

To do

- Handling of graph matching
- Rule selection heuristics
- Graph grammars for common data structures
- Tests with examples
- Logic
- Verification framework
- Implementation

Thank you for your attention!